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Mathematical expressions of entropy generation in a variable Viscosity MHD channel flow

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Abstract

The investigations are performed on thermodynamic second law on steady flow of an incompressible variable viscosity electrically conducting fluid in a channel with permeable walls using convective surface boundary conditions. The non-linear differential equations are solved analytically. Homotopy analysis method is used to determine the analytical expressions of dimensionless fluid velocity and dimensionless temperature profiles. We also derived the analytical expressions of the entropy generation rate and Bejan number. The graphical representations of temperature, velocity, entropy generation rate, Bejan number are also presented and discussed quantitatively. The Homotopy analysis method contains the convergence control parameter h_1 so it can be extended to solve the strongly non-linear boundary value problem in other MHD flow problem. The flow systems were controlled by the regulated values of thermophysical parameters. This method can be easily extended to solve the other non-linear initial and boundary value problems in physical, chemical and biological sciences.

Keywords: Permeable wall channel; Convective heat; Entropy generation; Bejan number; Homotopy analysis method.

1. Introduction

In recent days, most of the scientist and engineers pay attention to the study of MHD flow of conducting fluid through a permeable channel between two parallel plates. MHD means study of magnetic properties of electrically conducting fluids. Various branches of application in MHD flows such as MHD generators, MHD pump, accelerators, electrostatics precipitation, polymer technology, petroleum industries, purification of crude oil, plasma studies, nuclear reactors, the boundary control in the field of aerodynamics and blood flow problems [1].Couette flow occurs in fluid machinery involving moving parts and is especially important for hydrodynamic lubrication. Many researchers have discussed about the fluid flow on MHD problems under various situations [2-4]. On the generalized couette flow, the effect of magnetic field was analyzed by Agarwal[5]. Makinde and Onyejekwe [6] were investigated about the combined effects of variable velocity and electrical conductivity on MHD generalized couette flow and heat transfer.

Entropy generation is closely related to thermodynamic irreversibility, which is met in all heat transfer processes Bejan [7, 8] was the first to introduce the theoretical concept of entropy generation based on second law analysis. Entropy generation plays the main role in different fields like geothermal systems, electronic cooling, heat exchanges, etc. All thermal systems meet with entropy generation. The suction /injection on the boundary layer control are familiar in the field of aerodynamics and space science [9].O.D. Makinde, A.S. Eegunjobi [10], studied the combined effects of convective heating and suction/injection on the entropy generation in a steady flow of an incompressible viscous fluid through a channel with permeable walls.O.D. Makinde, A.S. Eegunjobi [11], investigated a numerical solution for the effect of convective heating on entropy generation in a channel with permeable walls. The combined effects of variable viscosity and asymmetric convective boundary conditions are extended on the entropy generation rate in MHD porous channel flow.

In this present work, the Entropy generation in a variable viscosity MHD channel flow is derived by using Homotopy analysis method. In the following sections, the model problem is formulated and analytically solved. The related results are shown graphically. We can apply this Homotopy analysis method to solve other non-linear differential equations.

2. Mathematical formulation of the problem

In this paper, consider a steady incompressible flow of an electrically conducting variable viscosity fluid between two fixed permeable parallel infinite plates of width h_1. The flow is fully developed and the edge effects are disregarded. Where B_0 denotes a constant magnetic field of strength which is imposed transversely in the y-direction. Here, both the electric field and Hall effect are not present ([12], [14]). We may assume that the applied magnetic field is to be strong enough so that the induced magnetic field due to the fluid motion is weak and can be ignored. We may assumed that the lower permeable plate denote fluid injection occurs, is convectively heated; while at the upper permeable plate both fluid suction and convective heat loss take place as shown in Figure 1.

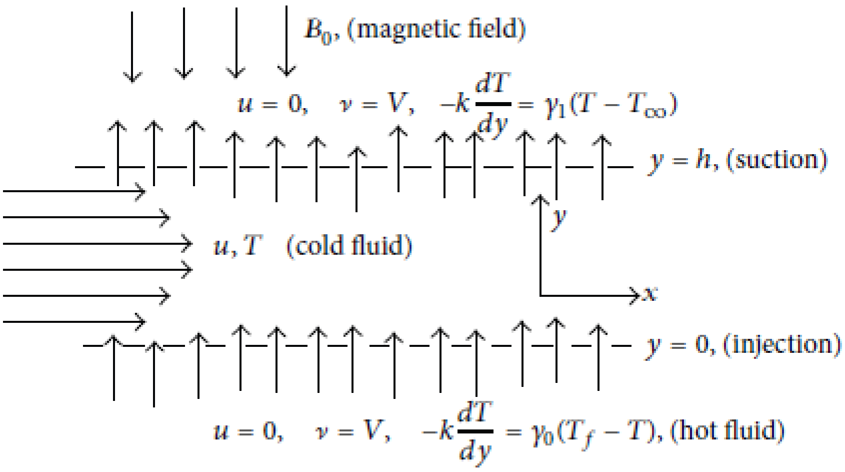

Fig.1: Schematic diagram

Therefore, the equations for the momentum and energy balance in one dimension can be written as follows [5-9]:

$$V\frac{du}{dy} = -\frac{1}{\rho}\frac{dP}{dx} + \frac{1}{\rho}\frac{d}{dx}\left(\bar{\mu}(T)\frac{du}{dy}\right) - \frac{\sigma B_0^{\,2} u}{\rho}$$

(1)

$$V\frac{dT}{dy} = \alpha\frac{d^2T}{dy^2} + \frac{\bar{\mu}(T)}{\rho c_P}\left(\frac{du}{dy}\right)^2 + \frac{\sigma B_0^{\,2} u^2}{\rho c_P}$$

(2)

The boundary conditions are as follows:

$$u(0) = 0, \quad -k\frac{dT}{dy}(0) = \gamma_0\left(T_f - T(0)\right)$$

$$u(h) = 0, \quad -k\frac{dT}{dy}(h) = \gamma_1\left(T(h) - T_\infty\right)$$

(3)

Where (x, y) denotes the axial and normal coordinates u denotes the velocity of the fluid, P denotes the fluid pressure, V denotes the uniform suction/injection velocity at the channel walls, γ_0 denotes the heat transfer coefficient at the lower plate, γ_1 denotes the heat transfer coefficient at the upper plate, α denotes the thermal diffusivity, ρ denotes the fluid density, σ denotes the fluid electrical conductivity, k denotes the thermal conductivity coefficient, C_P denotes the specific heat at constant pressure, T_f denotes the temperature, of the hot fluid at the lower permeable plate, T denotes the channel fluid temperature, and T_∞ is the ambient temperature above the upper plate. The temperature dependent viscosity $\bar{\mu}$ can be expressed as [9]

$$\bar{\mu}(T) = \mu_0 e^{-m(T-T_\infty)}$$

(4)

Where m denotes a viscosity variation parameter and μ_0 denotes the fluid dynamic viscosity at the ambient temperature. We introduce the following non dimensional quantities:

$$\eta = \frac{y}{h},\ \ \alpha = \frac{k}{\rho c_P},\ \ \ w = \frac{u}{V},$$

$$\theta = \frac{T - T_\infty}{T_f - T_\infty},\ \ X = \frac{x}{h},\ \ \bar{P} = \frac{ph}{\mu_0 V},$$

$$G = -\frac{\partial \bar{P}}{\partial X},\ \ \mu = \frac{\bar{\mu}}{\mu_0},\ \ v = \frac{\mu_0}{\rho},$$

(5)

Substituting the eqn. (5) into the eqns.(1)-(4), we obtain the following non-linear differential equations:

$$\frac{d^2 w}{d\eta^2} - \varepsilon \frac{d\theta}{d\eta}\frac{dw}{d\eta} - e^{\varepsilon\theta}\left(Re\frac{dw}{d\eta} + Ha\, w - G \right) = 0$$

(6)

$$\frac{d^2\theta}{d\eta^2} - Re\,Pr\frac{d\theta}{d\eta} + Ec\,Pr\,e^{-\varepsilon\theta}\left(\frac{dw}{d\eta}\right)^2 + Ec\,Pr\,Ha\,w^2 = 0$$

(7)

The corresponding boundary conditions are as follows:

$$w(0) = 0,\ \ \ \frac{d\theta}{d\eta}(0) = Bi_0\big(\theta(0) - 1\big),$$

$$w(1) = 0,\ \ \ \frac{d\theta}{d\eta}(1) = -Bi_1\theta(1)$$

(8)

$$Re = \frac{V\,h}{v}$$
$$Pr = \frac{v}{\alpha}$$
$$Ec = \frac{V^2}{c_p\,(T_f - T_\infty)}$$
$$Ha = \frac{\sigma B_0^{\ 2} h^2}{\mu_0} \qquad (9)$$
$$\varepsilon = m\,(T_f - T_\infty)$$
$$Bi_0 = \frac{\gamma_0 h}{k}$$
$$Bi_1 = \frac{\gamma_1 h}{k}$$

Where $\mathrm{Re}, \mathrm{Pr}, Ec, Ha, \varepsilon, Bi_0, Bi_1$ denote the Reynolds number, Prandtl number, Eckert number, Magnetic field parameter or square of Hartmann number, Variable viscosity parameter, Lower plate Biot number, Upper plate Biot number respectively and G denote the pressure gradient parameter.

It is important to note that $\varepsilon = 0$ corresponds to the case of constant viscosity conducting fluid. The exact solution of the eqn.(6) for the fluid velocity is possible under this constant viscosity scenario and we obtain

$$w(\eta) = \frac{G}{Ha}\left[1 + \frac{e^{\alpha\eta}(e^{\beta} - 1) - e^{\beta\eta}(e^{\alpha} - 1)}{e^{\alpha} - e^{\beta}}\right]$$

(10)

Here $\alpha = \left(\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\,Ha}\right)\big/2$ and $\beta = \left(Re - \sqrt{Re^2 + 4\,Ha}\right)\big/2$

(11)

In many engineering and industrial processes, entropy production destroys the available energy in the system. It is therefore imperative to determine the rate of entropy generation in a system, in order to optimize energy in the system for efficient operation in the system. The convection process in a channel is inherently irreversible and this causes continuous entropy generation. Wood [13] gave the local volumetric rate of entropy generation for a viscous incompressible conducting fluid in the presence of magnetic field as follows:

$$E_G = \frac{k}{T_\infty^{\ 2}}\left(\frac{dT}{dy}\right)^2 + \frac{\mu}{T_\infty}\left(\frac{du}{dy}\right)^2 + \frac{\sigma B_0^{\ 2}}{T_\infty}u^2$$

(12)

In an eqn. (11), the first term represents irreversibility due to heat transfer; the second term is entropy generation due to viscous dissipation, while the third term is local entropy generation due to the effect of the magnetic field (Joule heating or Ohmic heating). Using the eqn.(5), the dimensionless form of local entropy generation rate in an eqn.(10) is given as follows:

$$Ns = \frac{T_\infty{}^2 h^2 E_G}{k(T_f - T_\infty)^2} = \left(\frac{d\theta}{d\eta}\right)^2 + \frac{Br}{\Omega}\left[e^{-\varepsilon\theta}\left(\frac{dw}{d\eta}\right)^2 + Ha\ w^2\right]$$

(13)

Where $\Omega = {(T_f - T_\infty)}/{T_\infty}$ denotes the temperature difference parameter and $Br = Ec\,\mathrm{Pr}$ denotes the Brinkmann number. The Bejan number (Be) and the entropy generation rate are defined as

$$Be = \frac{N_1}{Ns} = \frac{1}{1+\Phi}$$

(14)

and

$$Ns = N_1 + N_2 \qquad (15)$$

where

$$N_1 = \left(\frac{d\theta}{d\eta}\right)^2$$

(16)

$$N_2 = \frac{Br}{\Omega}\left[e^{-\varepsilon\theta}\left(\frac{dw}{d\eta}\right)^2 + Ha\ w^2\right]$$

(17)

$$\Phi = \frac{N_2}{N_1}$$

(18)

Here N_1 denotes heat transfer irreversibility due to heat transfer, N_2 denotes fluid friction and magnetic field irreversibility, Φ denotes irreversibility ratio. The Bejan number (Be) as shown in (14) has a range of $0 \le Be \le 1$. If $Be = 0$ then the irreversibility is dominated by the combined effects of fluid friction and magnetic fields, but if $Be = 1$, then the irreversibility due to heat transfer dominates the flow system by the virtue of finite temperature differences.

3. Solution of the non-linear boundary value problems using the Homotopy analysis method

HAM is a non perturbative analytical method for obtaining series solutions to nonlinear equations and has been successfully applied to numerous problems in science and engineering [15-30]. In comparison with other perturbative and non perturbative analytical methods, HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter. Because of this, HAM has proved to be the most effective method for obtaining analytical solutions to highly non-linear differential equations. Previous applications of HAM have mainly focused on non-linear differential equations in which the non-linearity is a polynomial in terms of the unknown function and its derivatives. As seen in (1), the non-linearity present in electro hydrodynamic flow takes the form of a rational function, and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, HAM yields excellent results.

Liao [15-23] proposed a powerful analytical method for non-linear problems, namely the Homotopy analysis method. This method provides an analytical solution in terms of an infinite power series. However, there is a practical need to evaluate this solution and to obtain numerical values from the infinite power series. In order to investigate the accuracy of the Homotopy analysis method (HAM) solution with a finite number of terms, the system of differential equations were solved. The Homotopy analysis method is a good technique comparing to another perturbation method. The Homotopy analysis method contains the auxiliary parameter h_1, which provides us with a simple way to adjust and control the convergence region of solution series. Using this method, we can obtain the following solution to (6) and (7) (see Appendix B). The approximate analytical expressions of the eqns. (6) and (7) by using HAM are as follows:

$$w(\eta)=\frac{-G\eta^2}{2}+\frac{G}{2}\eta-h_1\left(\begin{array}{l}\frac{-c_2}{Re\,Pr}\left[\frac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]+\frac{c_2\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}+\frac{c_2\;G\,e^{Re\,Pr\,\eta}}{2(Re\,Pr)^2}-\frac{Re\,G\eta^3}{6}\\ +\frac{Re\;G\eta^2}{4}-\frac{Ha\;G\,\eta^4}{24}+\frac{Ha\;G\,\eta^3}{12}+\frac{c_3\eta^3}{6}-\frac{c_3\eta^2}{4}+\frac{c_4\eta^4}{12}-\frac{c_4\eta^3}{6}\\ +\frac{c_5\,G\eta^2}{2}+\frac{c_6\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}-\frac{c_6}{Re\,Pr}\left[\frac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]+\frac{c_6\;e^{Re\,Pr\,\eta}}{2(Re\,Pr)^2}\\ -\frac{c_7\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}+\frac{2c_7}{(Re\,Pr)^2}\left[\frac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]-\frac{2c_7\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^4}\\ -\frac{c_7}{Re\,Pr}\left[\frac{\eta^2\,e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{2\eta\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}+\frac{2\;e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}\right]-\frac{c_8\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\\ +\frac{c_7}{Re\,Pr}\left[\frac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]-(c_9+c_{10})\eta+c_9\end{array}\right)$$

(19)

$$\theta(\eta)=\left[\begin{array}{l} -c_1\varepsilon\left(e^{Re\,Pr}(Re\,Pr+Bi_1)-Bi_1e^{Re\,Pr\,\eta}\right) \\ -h_1\left(\begin{array}{l} \dfrac{Ec\,Pr}{Re\,Pr}\left[\begin{array}{l} \dfrac{G^2\eta}{4}+\dfrac{G^2\eta^3}{3}+\dfrac{G^2\eta^2}{Re\,Pr}+\dfrac{2G^2\eta}{(Re\,Pr)^2}-G^2\eta^2-\dfrac{2G^2\eta}{Re\,Pr} \\ +\varepsilon c_1 e^{Re\,Pr}(Re\,Pr+Bi_1)\left[\begin{array}{l} \dfrac{G^2\eta}{4}+\dfrac{G^2\eta^3}{3}+\dfrac{G^2\eta^2}{Re\,Pr}+ \\ \dfrac{2G^2\eta}{(Re\,Pr)^2}-G^2\eta^2-\dfrac{2G^2\eta}{Re\,Pr} \end{array}\right] \\ -3\,\varepsilon c_1 Bi_1 G^2\dfrac{e^{Re\,Pr\,\eta}}{4\,Re\,Pr}-2\,\varepsilon c_1 Bi_1 G^2\dfrac{e^{Re\,Pr\,\eta}\eta^2}{Re\,Pr} \\ +4\,\varepsilon c_1 Bi_1 G^2\dfrac{e^{Re\,Pr\,\eta}\eta}{(Re\,Pr)^2}-4\varepsilon c_1 Bi_1 G^2\dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^3} \\ +\left(4\,\varepsilon c_1 Bi_1 G^2-2\,\varepsilon c_1 Bi_1\dfrac{G^2}{Re\,Pr}\right)\left[\dfrac{e^{Re\,Pr\,\eta}\eta}{Re\,Pr}-\dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right] \\ +2\,\varepsilon c_1 Bi_1 G^2\dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2} \end{array}\right] \\ +\dfrac{Ec\,Pr\,Ha}{Re\,Pr}\left[\begin{array}{l} -\dfrac{3G^2\eta^2}{2(Re\,Pr)^2}-\dfrac{G^2\eta^3}{2\,Re\,Pr}+\dfrac{G^2\eta^2}{4\,Re\,Pr}+\dfrac{G^2\eta^4}{4\,Re\,Pr} \\ +\dfrac{2G^2\eta}{4(Re\,Pr)^2}+\dfrac{G^2\eta^3}{(Re\,Pr)^2}+\dfrac{G^2\eta^5}{20}\dfrac{G^2\eta^3}{12}-\dfrac{G^2\eta^4}{8} \end{array}\right]+c_{11}+c_{12}e^{Re\,Pr\,\eta} \end{array}\right) \end{array}\right]$$

(20)

$$Ns = N_1 + N_2$$

(21)

$$Be = \frac{N_1}{Ns}$$

(22)

where

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$$
N_1 = \left[\begin{aligned}
& - c_1 \varepsilon \left(e^{Re\,Pr} (Re\,Pr + Bi_1) - Re\,Pr\,Bi_1 e^{Re\,Pr\,\eta} \right) \\
& - h_1 \left(\begin{aligned}
& c_{12}\, Re\,Pr\, e^{Re\,Pr\,\eta} + \frac{Ec\,Pr}{Re\,Pr} \left[\begin{aligned}
& \frac{G^2}{4} + \frac{3G^2\eta^2}{3} + \frac{2G^2\eta}{Re\,Pr} + \frac{2G^2}{(Re\,Pr)^2} - 2G^2\eta - \frac{2G^2}{Re\,Pr} \\
& + \varepsilon c_1 e^{Re\,Pr} (Re\,Pr + Bi_1) \left[\begin{aligned} & \frac{G^2}{4} + \frac{3G^2\eta^2}{3} + \frac{2G^2\eta}{Re\,Pr} + \\ & \frac{2G^2}{(Re\,Pr)^2} - 2G^2\eta - \frac{2G^2}{Re\,Pr} \end{aligned} \right] \\
& - 3\,\varepsilon c_1 Bi_1 G^2 \frac{e^{Re\,Pr\,\eta}}{4} \\
& - 2\varepsilon c_1 Bi_1 G^2 \frac{(2e^{Re\,Pr\,\eta}\eta + \eta^2\, Re\,Pr\, e^{Re\,Pr\,\eta})}{Re\,Pr} \\
& + 4\varepsilon c_1 Bi_1 G^2 \frac{(e^{Re\,Pr\,\eta} + \eta\, Re\,Pr\, e^{Re\,Pr\,\eta})}{(Re\,Pr)^2} \\
& - 4\varepsilon c_1 Bi_1 G^2 \frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2} \\
& + (4\varepsilon c_1 Bi_1 G^2) \left[\frac{(e^{Re\,Pr\,\eta} + \eta\, Re\,Pr\, e^{Re\,Pr\,\eta})}{Re\,Pr} - \frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)} \right] \\
& - 2\varepsilon c_1 Bi_1 \frac{G^2}{Re\,Pr}) \left[\frac{e^{Re\,Pr\,\eta} + \eta\, Re\,Pr\, e^{Re\,Pr\,\eta}}{Re\,Pr} - \frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)} \right] \\
& + 2\varepsilon c_1 Bi_1 G^2 \frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)}
\end{aligned} \right] \\
& + \frac{Ec\,Pr\,Ha}{Re\,Pr} \left[\begin{aligned}
& \frac{3G^2\eta^2}{12} + \frac{5G^2\eta^4}{20} - \frac{4G^2\eta^3}{8} + \frac{2G^2\eta}{4\,Re\,Pr} + \frac{4G^2\eta^3}{4\,Re\,Pr} \\
& - \frac{3G^2\eta^2}{2\,Re\,Pr} + \frac{2\,G^2}{4(Re\,Pr)^2} + \frac{3G^2\eta^2}{(\,Re\,Pr)^2} - \frac{6\,G^2\eta}{2(Re\,Pr)^2}
\end{aligned} \right]
\end{aligned} \right)
\end{aligned} \right]^2
$$

(23)

$$N_2=\frac{Br}{\Omega}\left[\begin{aligned}&e^{-\varepsilon\theta}\left(-G\eta+\frac{G}{2}-h_1\left(\begin{aligned}&\frac{-c_2}{Re\,Pr}\left[\frac{(e^{Re\,Pr\,\eta}+\eta\,Re\,Pr\,e^{Re\,Pr\,\eta})}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)}\right]+\frac{c_2\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\\&+\frac{c_2\,G\,e^{Re\,Pr\,\eta}}{2(Re\,Pr)}-\frac{Re\,G\eta^2}{2}+\frac{Re\,G\eta}{2}-\frac{Ha\,G\eta^3}{6}+\frac{Ha\,G\eta^2}{4}\\&+\frac{c_3\eta^2}{2}-\frac{c_3\eta}{2}+\frac{c_4\eta^3}{3}-\frac{c_4\eta^2}{2}+c_5\,G\eta+\frac{c_6\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\\&-\frac{c_6}{Re\,Pr}\left[\frac{(e^{Re\,Pr\,\eta}+\eta\,Re\,Pr\,e^{Re\,Pr\,\eta})}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)}\right]+\frac{c_6\,e^{Re\,Pr\,\eta}}{2(Re\,Pr)}\\&+\frac{2c_7}{(Re\,Pr)^2}\left[\frac{(e^{Re\,Pr\,\eta}+\eta\,Re\,Pr\,e^{Re\,Pr\,\eta})}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)}\right]\\&-\frac{c_7}{Re\,Pr}\left[\begin{aligned}&\frac{(2e^{Re\,Pr\,\eta}\eta+\eta^2\,Re\,Pr\,e^{Re\,Pr\,\eta})}{Re\,Pr}\\&-\frac{2(e^{Re\,Pr\,\eta}+\eta\,Re\,Pr\,e^{Re\,Pr\,\eta})}{(Re\,Pr)^2}+\frac{2\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\end{aligned}\right]-\frac{2c_7\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}\\&-\frac{c_7\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}+\frac{c_7}{Re\,Pr}\left[\frac{(e^{Re\,Pr\,\eta}+\eta\,Re\,Pr\,e^{Re\,Pr\,\eta})}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)}\right]\\&-\frac{c_8\,e^{Re\,Pr\,\eta}}{(Re\,Pr)}-(c_9+c_{10})\end{aligned}\right)\right)^2\\&+Ha\left(\frac{-G\eta^2}{2}+\frac{G}{2}\eta-h_1\left(\begin{aligned}&\frac{-c_2}{Re\,Pr}\left[\frac{\eta\,e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]+\frac{c_2\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}+\frac{c_2\,G\,e^{Re\,Pr\,\eta}}{2(Re\,Pr)^2}\\&-\frac{Re\,G\eta^3}{6}+\frac{Re\,G\eta^2}{4}-\frac{Ha\,G\eta^4}{24}+\frac{Ha\,G\eta^3}{12}+\frac{c_3\eta^3}{6}-\frac{c_3\eta^2}{4}\\&+\frac{c_4\eta^4}{12}-\frac{c_4\eta^3}{6}+\frac{c_5\,G\eta^2}{2}+\frac{c_6\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}-\frac{c_6}{Re\,Pr}\left[\frac{\eta\,e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]\\&+\frac{c_6\,e^{Re\,Pr\,\eta}}{2(Re\,Pr)^2}+\frac{2c_7}{(Re\,Pr)^2}\left[\frac{\eta\,e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]-\frac{2c_7\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^4}\\&-\frac{c_7\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}-\frac{c_7}{Re\,Pr}\left[\frac{\eta^2\,e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{2\eta\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}+\frac{2\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}\right]\\&+\frac{c_7}{Re\,Pr}\left[\frac{\eta\,e^{Re\,Pr\,\eta}}{Re\,Pr}-\frac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right]-\frac{c_8\,e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}-(c_9+c_{10})\eta+c_9\end{aligned}\right)\right)^2\end{aligned}\right]$$

(24)

$$c_1=\frac{Bi_0}{\left[Bi_1(Bi_0-Re\,Pr)-Bi_0\,e^{Re\,Pr}(Re\,Pr+Bi_1)\right]}$$

(25)

$$c_2=\varepsilon c_1\,Bi_1\,Re\,Pr$$

(26)

$$c_3=\varepsilon c_1\,e^{Re\,Pr}(Re\,Pr+Bi_1)\,Re\,G$$

(27)

$$c_4=\varepsilon c_1\,e^{Re\,Pr}(Re\,Pr+Bi_1)\,Ha\,\frac{G}{2}$$

(28)

$$c_5=\varepsilon c_1\,e^{Re\,Pr}(Re\,Pr+Bi_1)\,G$$

(29)

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$$c_6 = \varepsilon c_1\, Bi_1\, Re\, G$$
(30)

$$c_7 = \varepsilon c_1\, Bi_1\, Ha \frac{G}{2}$$
(31)

$$c_8 = \varepsilon c_1\, Bi_1\, G$$
(32)

$$c_9 = \frac{-2c_2}{(Re\,Pr)^3} - \frac{c_2\,G}{2(Re\,Pr)^2} - \frac{2c_6}{(Re\,Pr)^3} - \frac{c_6}{2(Re\,Pr)^2} + \frac{4c_7}{(Re\,Pr)^4} + \frac{2c_7}{(Re\,Pr)^4} + \frac{2c_7}{(Re\,Pr)^3} + \frac{c_8}{(Re\,Pr)^2}$$
(33)

$$c_{10} = \left[\begin{array}{l} \frac{-c_2}{Re\,Pr}\left(\frac{e^{Re\,Pr}}{Re\,Pr} - \frac{e^{Re\,Pr}}{(Re\,Pr)^2}\right) + \frac{c_2\,e^{Re\,Pr}}{(Re\,Pr)^3} + \frac{c_2\,G\,e^{Re\,Pr}}{2(Re\,Pr)^2} - \frac{Re\,G}{6} + \frac{Re\,G}{4} - \frac{G\,Ha}{24} + \frac{G\,Ha}{12} \\ \quad -\frac{G}{2} + \frac{c_3}{6} - \frac{c_3}{4} + \frac{c_4}{12} - \frac{c_4}{6} + \frac{c_5\,G}{2} - \frac{c_6}{Re\,Pr}\left(\frac{e^{Re\,Pr}}{Re\,Pr} - \frac{e^{Re\,Pr}}{(Re\,Pr)^2}\right) + \frac{c_6\,e^{Re\,Pr}}{(Re\,Pr)^3} + \frac{c_6\,e^{Re\,Pr}}{2(Re\,Pr)^2} \\ \quad -\frac{c_7}{Re\,Pr}\left(\frac{e^{Re\,Pr}}{Re\,Pr} - \frac{2e^{Re\,Pr}}{(Re\,Pr)^2} + \frac{2e^{Re\,Pr}}{(Re\,Pr)^3}\right) + \frac{2c_7}{(Re\,Pr)^2}\left(\frac{e^{Re\,Pr}}{Re\,Pr} - \frac{e^{Re\,Pr}}{(Re\,Pr)^2}\right) \\ \quad + \frac{c_7}{Re\,Pr}\left(\frac{e^{Re\,Pr}}{Re\,Pr} - \frac{e^{Re\,Pr}}{(Re\,Pr)^2}\right) + \frac{c_7\,e^{Re\,Pr}}{(Re\,Pr)^3} - \frac{c_8\,e^{Re\,Pr}}{(Re\,Pr)^2} \end{array}\right]$$
(34)

$$c_{11} = \frac{-\left(Bi_0 - \frac{1}{Re\,Pr}\right)\left(Bi_1 f_1 - Bi_0(-f_2 - Bi_1 f_3)\right)}{Bi_0\left[Bi_1\left(Bi_0 - \frac{1}{Re\,Pr}\right) - Bi_0\left(Bi_1 e^{Re\,Pr} + \frac{e^{Re\,Pr}}{Re\,Pr}\right)\right]} + f_1$$
(35)

$$c_{12} = \frac{\left(Bi_1 f_1 - Bi_0(-f_2 - Bi_1 f_3)\right)}{\left[Bi_1\left(Bi_0 - \frac{1}{Re\,Pr}\right) - Bi_0\left(Bi_1 e^{Re\,Pr} + \frac{e^{Re\,Pr}}{Re\,Pr}\right)\right]}$$
(36)

$$f_1 = \left(\begin{array}{l} \frac{Ec\,Pr\,G}{(Re\,Pr)^2} + \varepsilon\,G\,c_1\,e^{Re\,Pr}(Re\,Pr + Bi_1)\,Ec\,Pr\left(\frac{1}{(Re\,Pr)^2}\right) + \frac{Ec\,Pr\,G}{2\,Re\,Pr} + \frac{3\,Ec\,Pr\,Ha\,G^2}{(Re\,Pr)^4} \\ \quad + \frac{Ec\,Pr\,\varepsilon\,G\,c_1\,Bi_1}{2}\left[\frac{1}{Re\,Pr} - \frac{1}{(Re\,Pr)^3}\right] + \frac{6\,Ec\,Pr\,Ha\,G^2}{(Re\,Pr)^5} + \frac{Ec\,Pr\,Ha\,G^2}{2(Re\,Pr)^3} \end{array}\right)$$
(37)

$$f_2 = \left(\begin{array}{l} Ec\,Pr\,G\left[\frac{-2}{Re\,Pr}+\frac{1}{(Re\,Pr)^2}\right]+\varepsilon\,G\,c_1\,e^{Re\,Pr}(Re\,Pr+Bi_1)\,Ec\,Pr\left(\frac{-2}{Re\,Pr}+\frac{1}{(Re\,Pr)^2}\right) \\ +\frac{Ec\,Pr\,G}{2\,Re\,Pr}+\frac{Ec\,Pr\,\varepsilon\,G\,c_1\,Bi_1}{2}\left[\frac{e^{Re\,Pr}}{(Re\,Pr)^2}+\frac{e^{Re\,Pr}}{Re\,Pr}-\frac{e^{Re\,Pr}}{(Re\,Pr)^3}\right] \\ -\frac{Ec\,Pr\,Ha\,G^2}{4}\left[\frac{-1}{Re\,Pr}+\frac{4}{(Re\,Pr)^2}-\frac{12}{(Re\,Pr)^3}+\frac{24}{(Re\,Pr)^4}-\frac{24}{(Re\,Pr)^5}\right] \\ -\frac{Ec\,Pr\,Ha\,G^2}{4}\left[\frac{-1}{Re\,Pr}+\frac{2}{(Re\,Pr)^2}-\frac{2}{(Re\,Pr)^3}\right] \\ +\frac{Ec\,Pr\,Ha\,G^2}{2}\left[\frac{-1}{Re\,Pr}+\frac{3}{(Re\,Pr)^2}-\frac{6}{(Re\,Pr)^3}+\frac{6}{(Re\,Pr)^4}\right] \end{array} \right) \quad (38)$$

$$f3 = \left(\begin{array}{l} Ec\,\mathrm{Pr}\,G\left[\frac{-1}{\mathrm{Re\,Pr}}+\frac{1}{(\mathrm{Re\,Pr})^2}\right]+\varepsilon\,G\,c_1\,e^{\mathrm{Re\,Pr}}(\mathrm{Re\,Pr}+Bi_1)\,Ec\,\mathrm{Pr}\left(\frac{-1}{\mathrm{Re\,Pr}}+\frac{1}{(\mathrm{Re\,Pr})^2}\right) \\ +\frac{Ec\,\mathrm{Pr}\,G}{2\,\mathrm{Re\,Pr}}+\frac{Ec\,\mathrm{Pr}\,\varepsilon\,G\,c_1\,Bi_1}{2}\left[\frac{e^{\mathrm{Re\,Pr}}}{\mathrm{Re\,Pr}}-\frac{e^{\mathrm{Re\,Pr}}}{(\mathrm{Re\,Pr})^2}\right] \\ -\frac{Ec\,\mathrm{Pr}\,Ha\,G^2}{4}\left[\frac{-1}{5\,\mathrm{Re\,Pr}}+\frac{1}{(\mathrm{Re\,Pr})^2}-\frac{4}{(\mathrm{Re\,Pr})^3}+\frac{12}{(\mathrm{Re\,Pr})^4}-\frac{24}{(\mathrm{Re\,Pr})^5}\right] \\ -\frac{Ec\,\mathrm{Pr}\,Ha\,G^2}{4}\left[\frac{-1}{3\,\mathrm{Re\,Pr}}+\frac{1}{(\mathrm{Re\,Pr})^2}-\frac{2}{(\mathrm{Re\,Pr})^3}\right] \\ +\frac{Ec\,\mathrm{Pr}\,Ha\,G^2}{2}\left[\frac{-1}{4\,\mathrm{Re\,Pr}}+\frac{3}{(\mathrm{Re\,Pr})^2}-\frac{3}{(\mathrm{Re\,Pr})^3}+\frac{6}{(\mathrm{Re\,Pr})^4}\right] \end{array} \right)$$

(39)

4. Results and Discussion

Figure1 shows the schematic diagram of MHD flow problem. Figure 2 (a)-(g) represents the dimensionless velocity profile $w(\eta)$ versus the dimensionless transverse coordinate η. From Fig.2 (a), it is clear that when square of Hartmann number Ha increases the corresponding dimensionless velocity profiles decreases in some fixed values of the other dimensionless parameters $\mathrm{Re}, Ec, \mathrm{Pr}, \varepsilon, Bi_0, Bi_1, G\, and\ h_1$. From Fig. 2(b), it is inferred that when the variable viscosity ε increases the corresponding dimensionless fluid velocity profiles also increases in some fixed values of the other dimensionless parameters. From Fig. 2(c) it is observed that when the Eckert number Ec increases the corresponding dimensionless fluid velocity profiles increases in some fixed values of the other dimensionless parameters. From Fig.2 (d) it is noted that when the Reynolds number Re increases the corresponding dimensionless velocity profiles decreases in some fixed values of the other dimensionless parameters. From Fig. 2(e) it is depict that when the Prandtl number Pr increases the corresponding dimensionless velocity profiles increases in some fixed values of the other dimensionless parameters. From Fig. 2(f) it is noted that when the lower plate Biot number Bi_0 increases the corresponding dimensionless velocity profiles increases in some fixed values of the other dimensionless parameters. From Fig. 2(g) it is inferred that when the upper plate

Biot number Bi_1 increases the corresponding dimensionless fluid velocity profiles increases in some fixed values of the other dimensionless parameters.

Figure 3(a)-(g) represents the dimensionless transverse coordinates versus dimensionless fluid temperature profiles. From Fig.3 (a), it is clear that when the square of Hartmann number increases the corresponding dimensionless fluid temperature profiles decreases in some fixed values of the other parameters. In Figure3(b) it is noted that when variable viscosity increases the corresponding dimensionless fluid temperature profiles increases in some fixed values of the other parameters. From Figure 3(c) it is observed that when the Reynolds number increases the Dimensionless fluid temperature profiles increases. In Figure 3(d), it is noted that when prandtl number increases the Dimensionless fluid temperature profiles increases. From Figure 3(e) it is inferred that the Eckert number increases the Dimensionless fluid temperature profiles increases. From Figure 3(f) it is observed that the lower plate Biot number Bi_0 increases the Dimensionless fluid temperature profiles increases. From Figure 3(g) it is noted that the upper plate Biot number Bi_1 increases the Dimensionless fluid temperature profiles decreases.

Figure 4 (a)-(g) represents the Entropy generation rate Ns versus dimensionless transverse coordinate η . Fig.4 (a) exposes the effect of increasing square of Hartmann number Ha on entropy generation rate Ns. When Ha increases, the entropy generation Ns decreases at the walls and increases at the centerline region of the channel. In the meantime, it is remarkable to note that two points exist, that is, $\eta = 0.2 \, and \, \eta = 0.8$ within the flow field where the entropy generation rate Ns is not affected by increasing Ha in some fixed values of the other parameters $Ec, \Pr, \varepsilon, \mathrm{Re}, Br\Omega^{-1}, G, Bi_0, Bi_1 \, and \, h_1$. From Fig.4 (b) it is clear that entropy production is improved with increasing suction (Re) at the upper wall region, while a decrease in the entropy generation occurs at the lower wall with increasing fluid injection. From Fig. 4(c), it is noted that as the fluid viscosity decreases with increasing values of ε the entropy generation rate Ns decreases at both walls and increases towards the channel centerline. A similar trend of entropy generation rate Ns is observed with increasing values of group parameter $Br\Omega^{-1}$ and constant pressure gradient (G) as demonstrated in Fig. 4(d) and Fig. 4(e). From Fig. 4(f) and Fig. 4(g) the entropy generation rate increases with combined increase in convective heating at the lower wall and convective cooling at the upper wall that is, as $Bi_0 \, and \, Bi_1$ increase.

Figure 5 represents the Bejan number Be versus dimensionless transverse coordinate η .the Bejan number is highest along the channel centerline region with irreversibility due to heat transfer dominating the flow, while near the channel walls the fluid friction and magnetic field irreversibility dominate. From Fig.5, (a) it is noted that an increase in Ha results in a decrease of Be along the channel centerline in some fixed values of the other parameters $Ec, \Pr, \mathrm{Re}, Br\Omega^{-1}, G, \varepsilon, Bi_0, Bi_1 \, and \, h_1$ .From Fig. 5(b) the Bejan number Be decreases near the lower wall due to injection and increases toward the upper wall due to suction. From Fig. 5(c)-(d) it is depict that decrease in Bejan number with increasing parameter values of ε and $Br\Omega^{-1}$ due to a decrease in fluid viscosity and an increase in viscous dissipation irreversibility. From Fig. 5(e)-(f) it is clear that an increase in the dominant influence of heat transfer irreversibility is observed as the parameter values of $Bi_0 \, and \, Bi_1$ increase, consequently, the Bejan number Be increases. Hence, the convective thermal boundary conditions enhance the dominant effects of heat transfer irreversibility on the flow system. From Fig. 5(g) it is inferred that an increase in the pressure gradient parameter causes a decrease in the Bejan number within the channel leading to an increase in the irreversibility due to fluid friction. From Fig. 5(h) it is note that the Bejan number slightly decreases at the lower plate and slightly increases at the upper plate with increasing Prandtl number. The Table.1 represents the comparison between the exact solution and the HAM solution for the velocity profiles at variable viscosity value is zero.

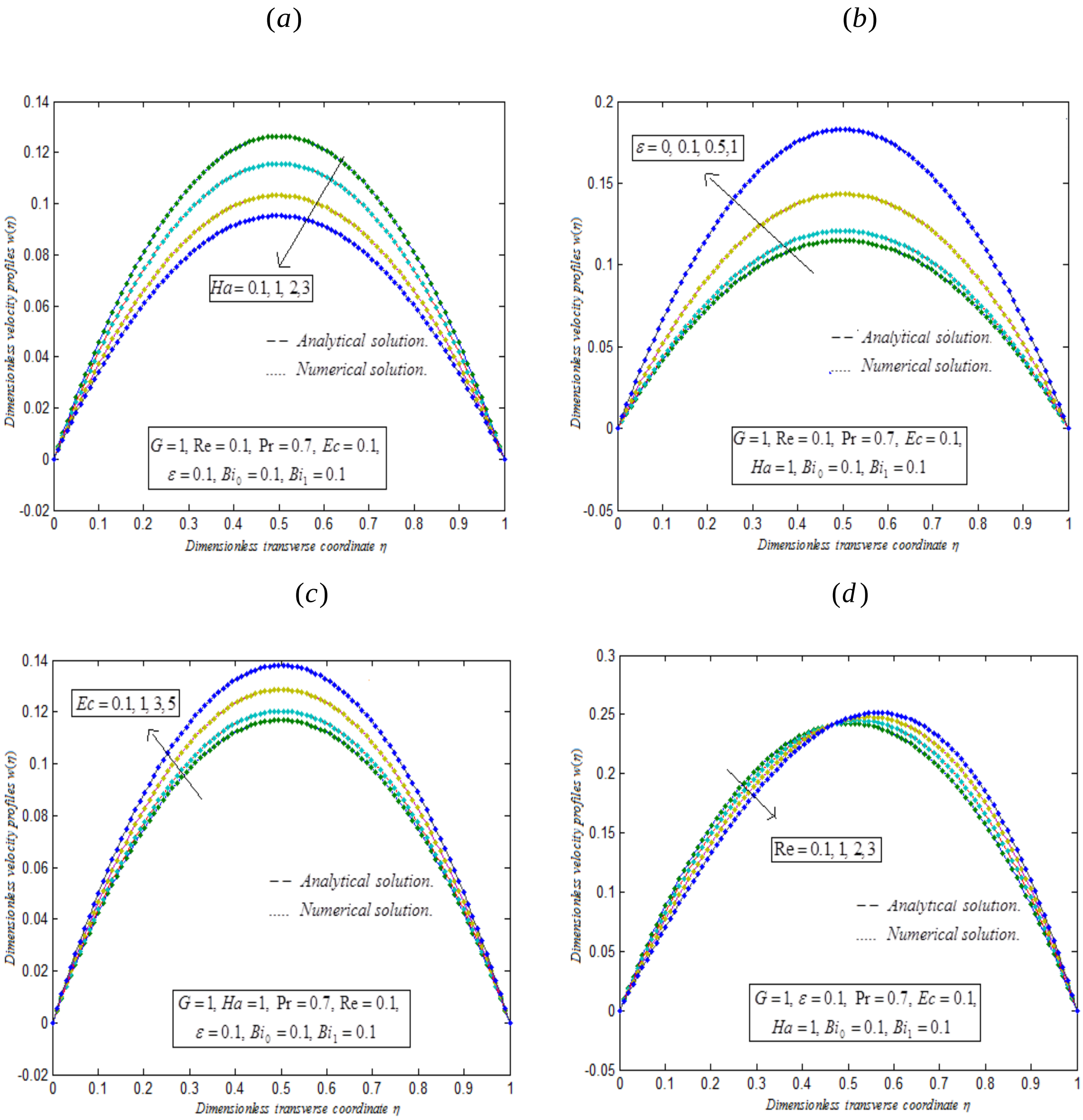
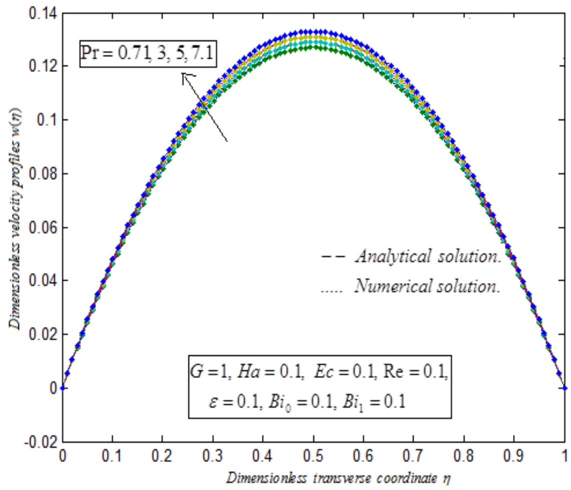
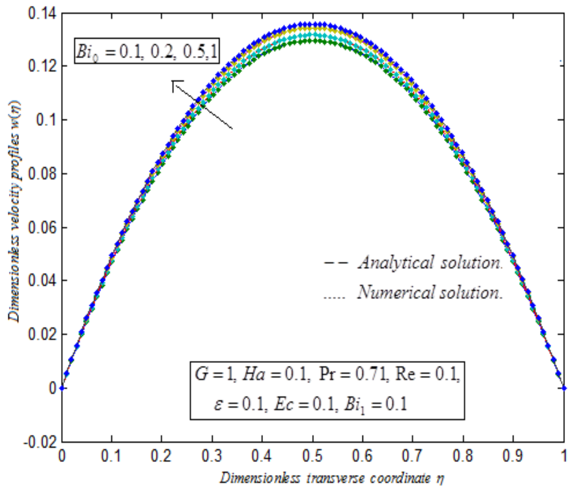


Fig.2 (a)-(d): Dimensionless fluid velocity profiles $w(\eta)$ versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (19) for various values of the dimensionless parameter ε, Ha, Ec, Re and in some fixed values of the other dimensionless parameters, when (a) $Ec, \Pr, \mathrm{Re}, G, \varepsilon, Bi_0, Bi_1, h_1 = -0.15525$, (b) $Ha, Ec, \Pr, \mathrm{Re}, G, Bi_0, Bi_1, h_1 = -0.15525$ (c) $Ha, \Pr, \mathrm{Re}, G, \varepsilon, Bi_0, Bi_1, h_1 = -0.15525$ and (d) $Ha, Ec, \Pr, G, \varepsilon, Bi_0, Bi_1, h_1 = -0.15525$.

(e) (f)





(g)

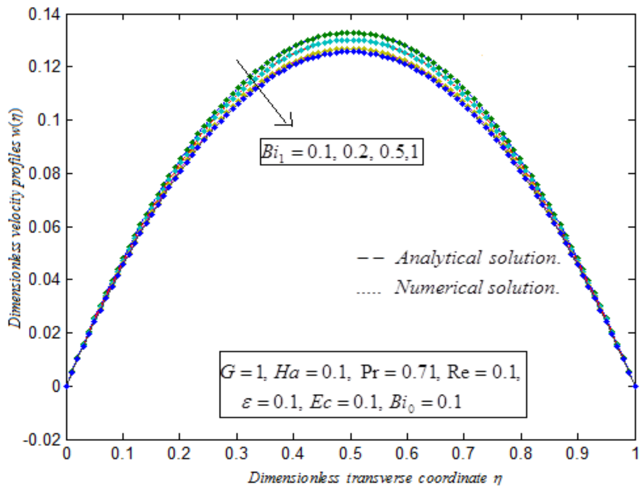


Fig.2(e)-(g): Dimensionless fluid velocity profiles $w(\eta)$ versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (19) for various values of the dimensionless parameter $\Pr, Bi_0, Bi_1$ and in some fixed values of the other dimensionless parameters, when (e) $Ha, Ec, \mathrm{Re}, G, \varepsilon, Bi_0, Bi_1, h_1 = -0.15525$, (f) $Ha, Ec, \Pr, \mathrm{Re}, G, \varepsilon, Bi_1, h_1 = -0.15525$ (g) $Ha, Ec, \Pr, \mathrm{Re}, G, \varepsilon, Bi_0, h_1 = -0.15525$.

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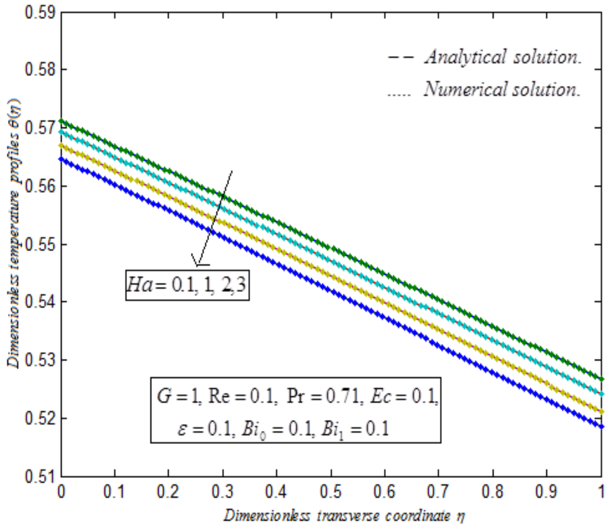
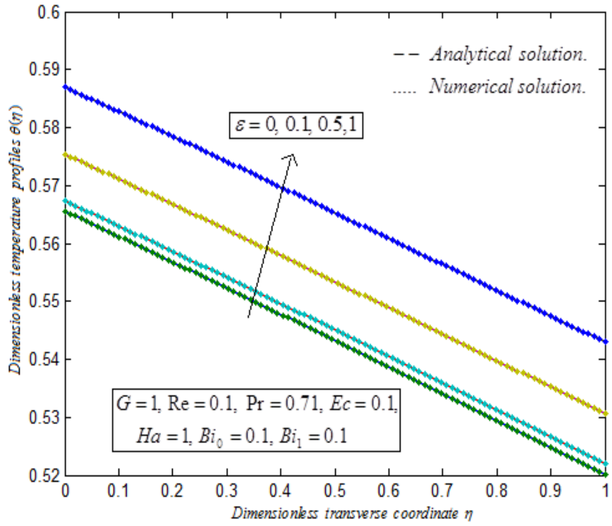
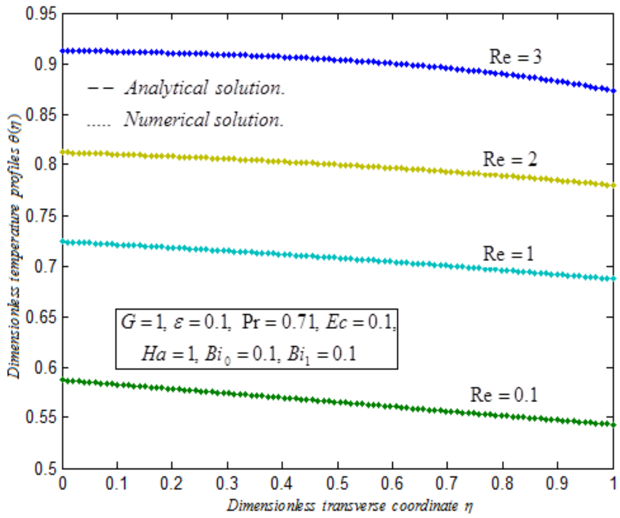
(a) (b)





(c) (d)



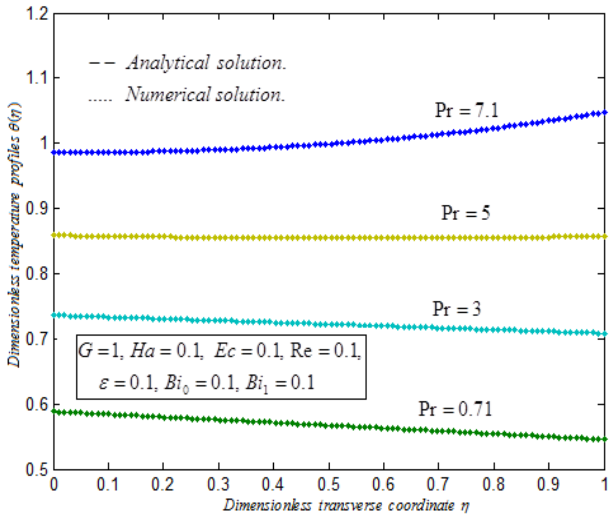
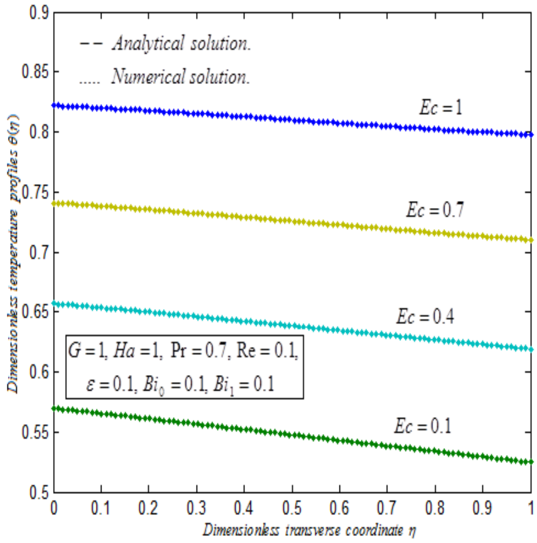
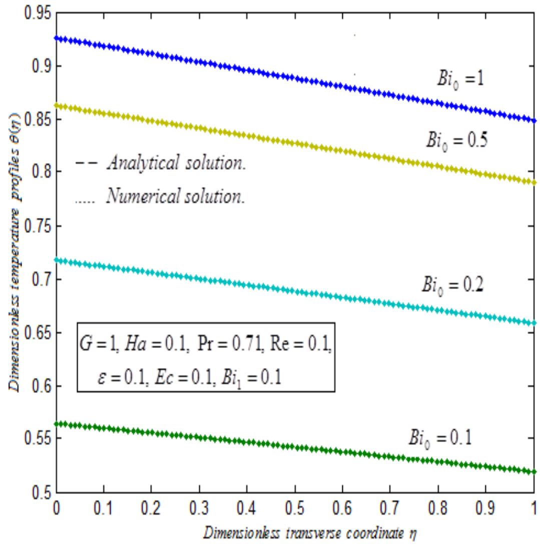


Fig.3(a)-(d): Dimensionless fluid temperature profiles $\theta(\eta)$ versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (20) for various values of the dimensionless parameter $Ha, \varepsilon, \mathrm{Re}, \mathrm{Pr}$ and in some fixed values of the other dimensionless parameters, when (a) $Ec, \mathrm{Pr}, \mathrm{Re}, G, \varepsilon, Bi_0, Bi_1, h_1 = 0.002519$, (b) $Ha, Ec, \mathrm{Pr}, \mathrm{Re}, G, Bi_0, Bi_1, h_1 = 0.002519$ (c) $Ha, \mathrm{Pr}, Ec, G, \varepsilon, Bi_0, Bi_1, h_1 = 0.002519$ and (d) $Ha, Ec, \mathrm{Re}, G, \varepsilon, Bi_0, Bi_1, h_1 = 0.002519$.

(e) (f)





(g)

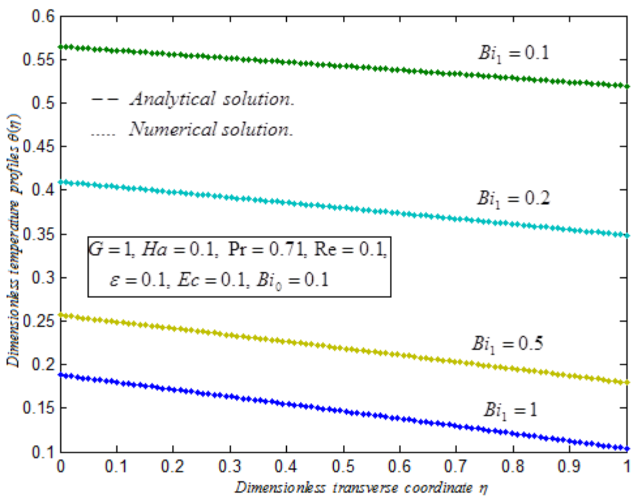
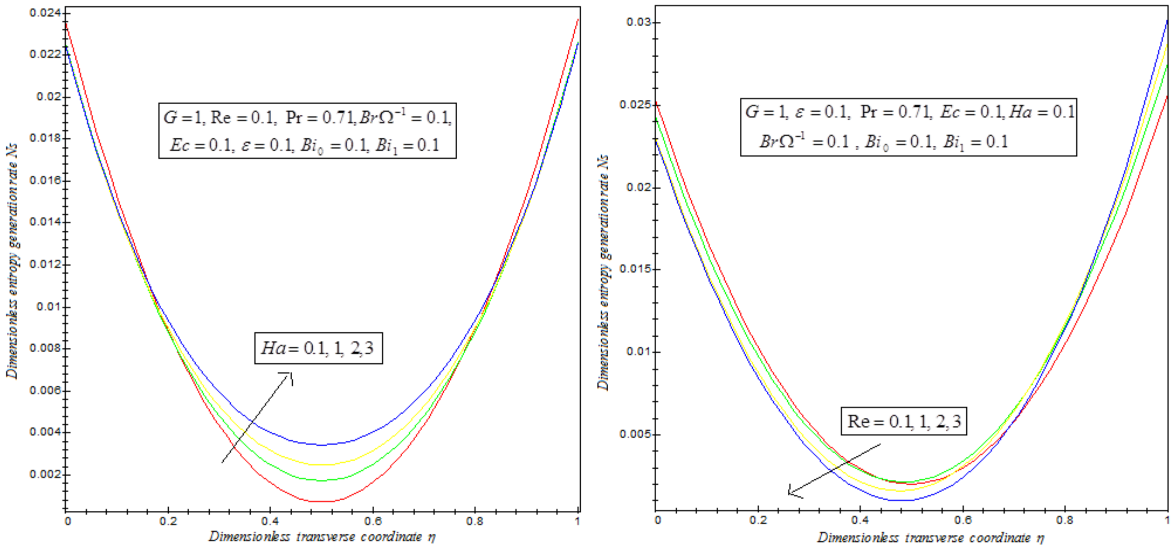


Fig.3(e)-(g): Dimensionless fluid temperature profiles $\theta(\eta)$ versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (20) for various values of the dimensionless parameter Ec, Bi_0, Bi_1 and in some fixed values of the other dimensionless parameters, when (e) $Ha, \Pr, \mathrm{Re}, G, \varepsilon, Bi_0, Bi_1, h_1 = 0.002519$, (f) $Ha, Ec, \Pr, \varepsilon, \mathrm{Re}, G, Bi_1, h_1 = 0.002519$ (g) $Ha, \Pr, Ec, \mathrm{Re}, G, \varepsilon, Bi_0, h_1 = 0.002519$.

(a) (b)



(c) (d)

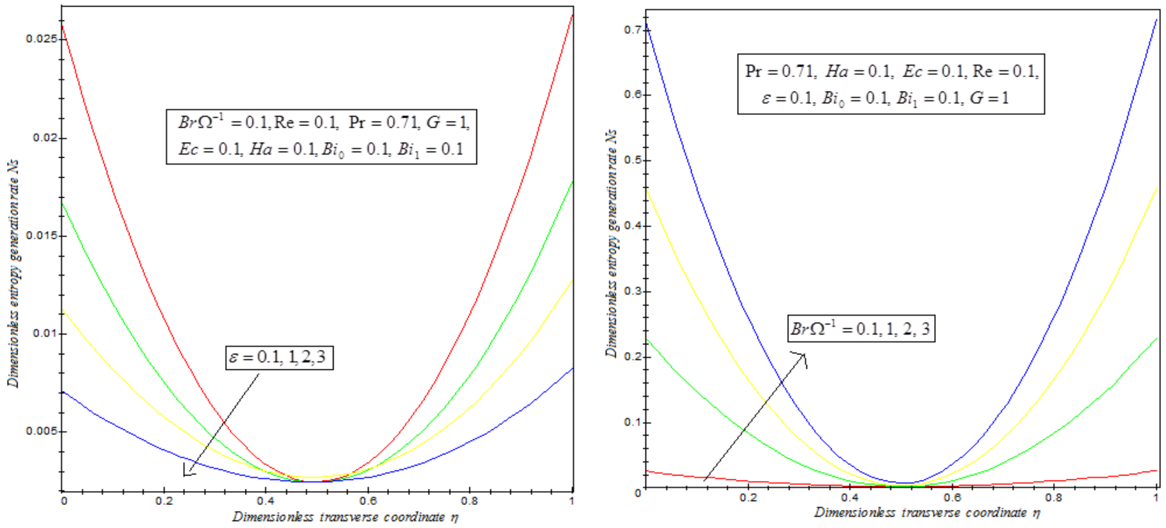
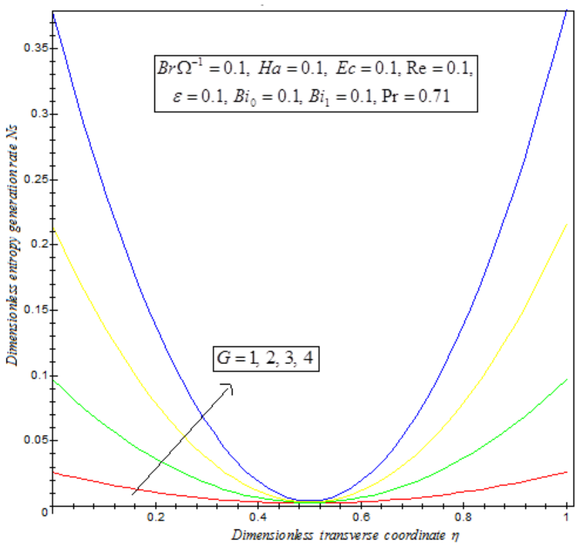
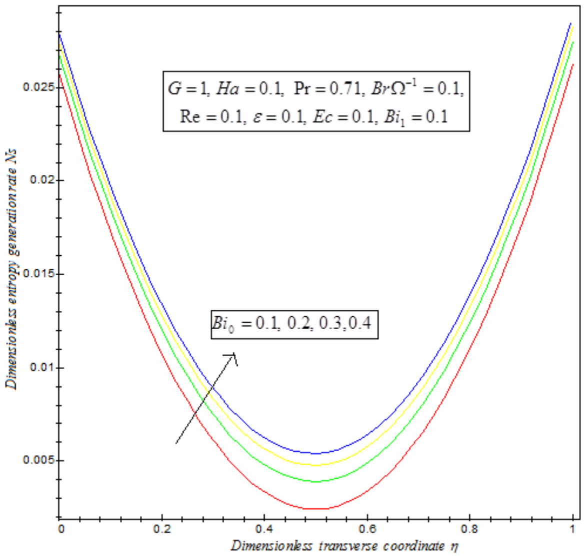


Fig.4(a)-(d): Dimensionless entropy generation rate Ns versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (21) for various values of the dimensionless parameter $Ha, \varepsilon, \mathrm{Re}, Br\Omega^{-1}$ and in some fixed values of the other dimensionless parameters, when (a) $Ec, \mathrm{Pr}, \mathrm{Re}, G, \varepsilon, Br\Omega^{-1}, Bi_0, Bi_1, h_1 = 0.000381$, (b) $Ha, Ec, \mathrm{Pr}, \mathrm{Re}, G, Bi_0, Bi_1, Br\Omega^{-1}$, $h_1 = 0.000381$ (c) Ha, $\mathrm{Pr}, Ec, G, \varepsilon, Bi_0, Bi_1, Br\Omega^{-1}, h_1 = 0.000381$ and (d) Ha, Ec, Re, G $\mathrm{Pr}, \varepsilon, Bi_0, Bi_1, h_1 = 0.000381$.

(e) (f)





(g)

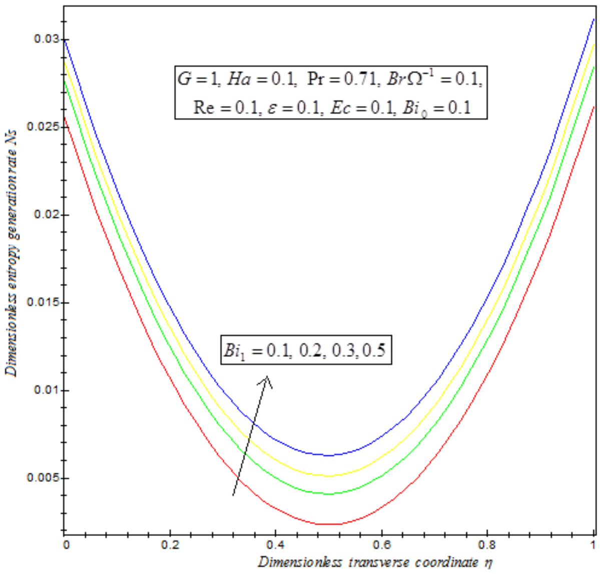
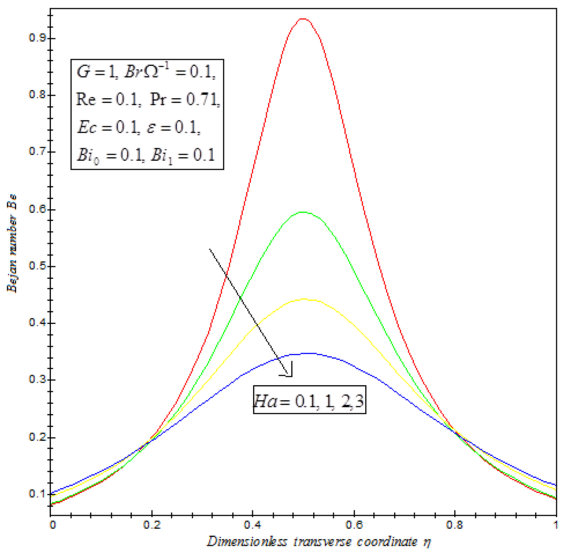
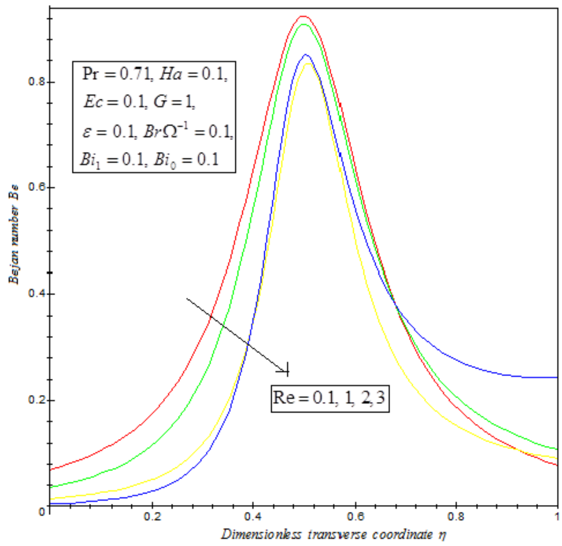


Fig.4(e)-(g): Dimensionless entropy generation rate Ns versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (21) for various values of the dimensionless parameter Bi_0, G, Bi_1 and in some fixed values of the other dimensionless parameters, when (e) $Ha, \mathrm{Pr}, \mathrm{Re}, G, \varepsilon, Ec, Bi_1, Br\Omega^{-1}, h_1 = 0.000381$, (f) $Ha, Ec, \mathrm{Pr}, \varepsilon, \mathrm{Re}, Bi_0$, $Bi_1, Br\Omega^{-1}$, $h_1 = 0.000381$ (g) Ha, $\mathrm{Pr}, Ec, \mathrm{Re}, G, \varepsilon, Bi_0, Br\Omega^{-1}, h_1 = 0.000381$.

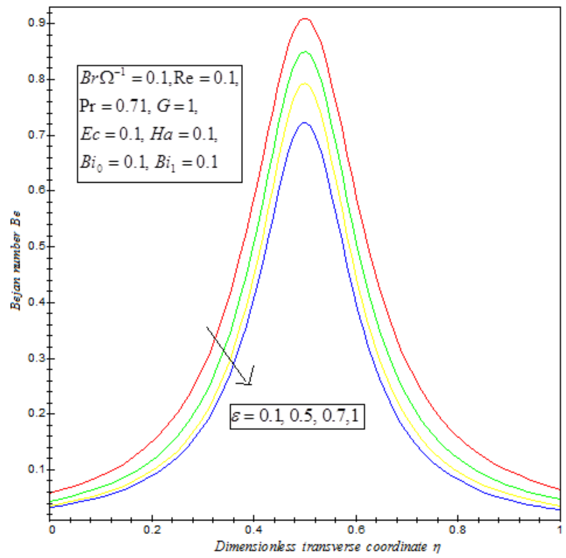
(a) (b)



(c)



(d)



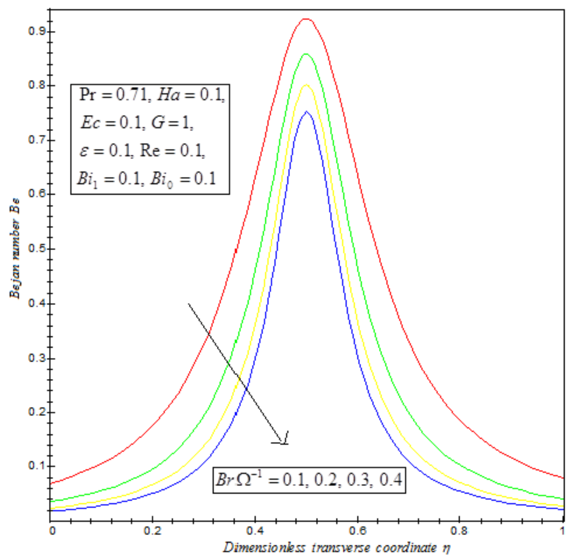


Fig.5 (a)-(d): Dimensionless Bejan number Be versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (22) for various values of the dimensionless parameter $Ha, \varepsilon, \mathrm{Re}, Br\Omega^{-1}$ and in some fixed values of the other dimensionless parameters, when (a) $Ec, \mathrm{Pr}, \mathrm{Re}, G, \varepsilon, Br\Omega^{-1}, Bi_0, Bi_1, h_1 = -0.003115$, (b) $Ha, \mathrm{Pr}, Ec, G, \varepsilon, Bi_0, Bi_1, Br\Omega^{-1}$, $h_1 = -0.003115$ (c) $Ha, Ec, \mathrm{Pr}, \mathrm{Re}, G, Bi_0, Bi_1, Br\Omega^{-1}$, $h_1 = -0.003115$ and (d) Ha, Ec, Re, G $\mathrm{Pr}, \varepsilon, Bi_0, Bi_1, h_1 = -0.003115$.

(e) (f)

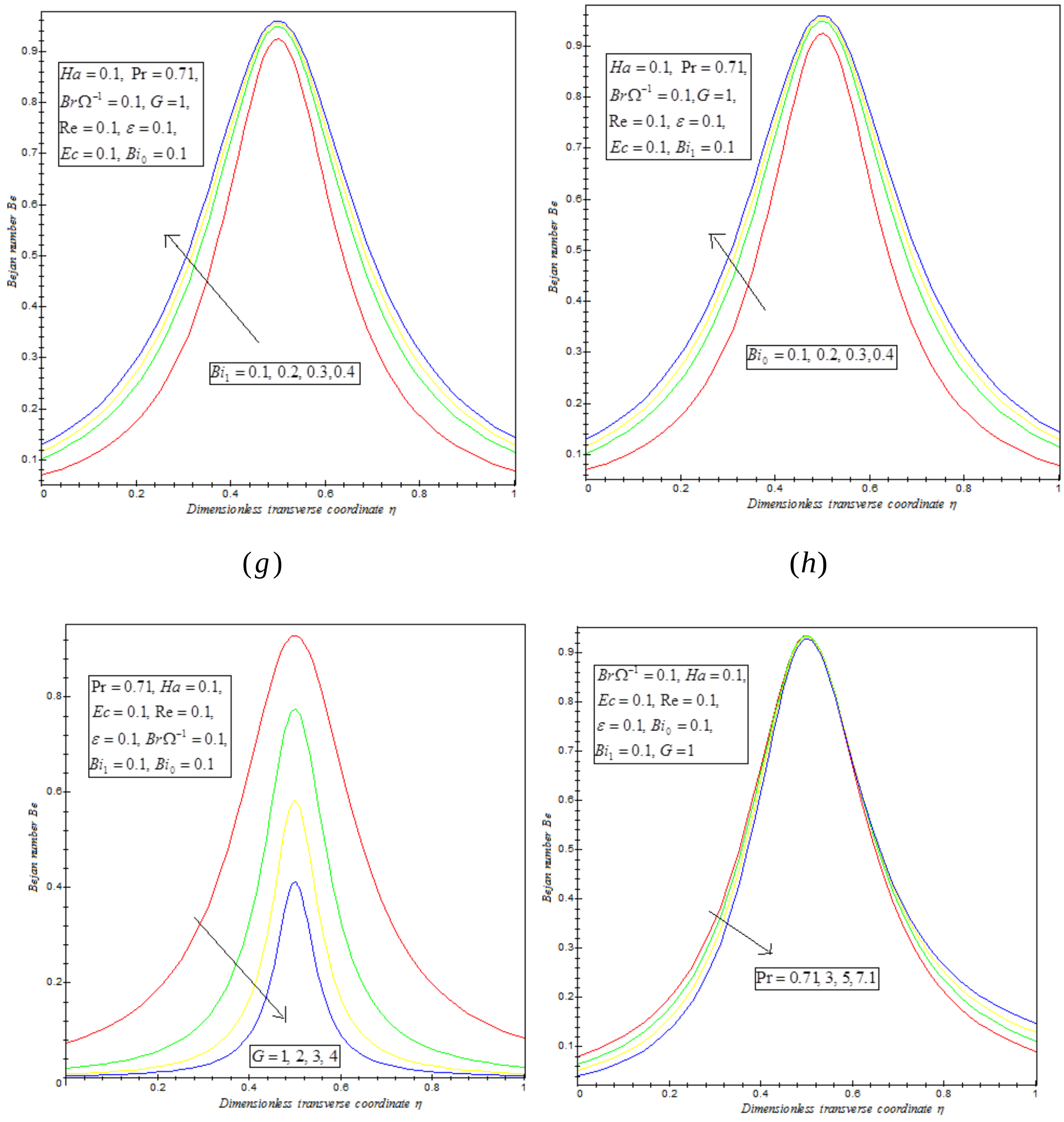


Fig.5 (e)-(h): Bejan number Be versus dimensionless transverse coordinate (η). The curves are plotted using the eqn. (22) for various values of the dimensionless parameter $Bi_1, Bi_0, G, \Pr$ and in some fixed values of the other dimensionless parameters, when (e) $Ec, \Pr, \mathrm{Re}, G, \varepsilon, Br\Omega^{-1}, Bi_0, Ha, h_1 = -0.003115$, (f) $Ha, Ec, \Pr, \mathrm{Re}, G, \varepsilon, Bi_1, Br\Omega^{-1}$, $h_1 = -0.003115$ (g) $Ha, \Pr, Ec, \mathrm{Re}, \varepsilon, Bi_0, Bi_1, Br\Omega^{-1}, h_1 = -0.003115$ and (h) Ha, Ec, Re, G $Br\Omega^{-1}, \varepsilon, Bi_0, Bi_1, h_1 = -0.003115$.

Table.1: This table showing the comparison of the exact and HAM solution of dimensionless fluid velocity profile for $\mathrm{Re}=1, Ha=1, G=1$ *and* $\varepsilon=0$.

η	*Exact solution* $(w(\eta))$	*HAM solution* $(w(\eta))$ $h_1=0.09162$
0	0	0
0.1	0.035822	0.035822
0.2	0.065264	0.065264
0.3	0.087963	0.087963
0.4	0.103449	0.103449
0.5	0.1111279	0.1111279
0.6	0.110257	0.110257
0.7	0.099930	0.099930
0.8	0.079042	0.079042
0.9	0.046264	0.046264
1.0	0	0

5. Conclusion

In this paper, second law analysis for MHD permeable channel flow with variable electrical conductivity is carried out. The velocity and temperature profiles are obtained analytically and also used to determine the entropy generation number and Bejan number. The graphical results presents the effects of the suction/injection Reynolds number, Magnetic field parameter, Eckert number, Prandtl number and the pressure gradient parameter on velocity, temperature profiles. The entropy generation rate and Bejan number are also presented by using the influences of these parameters and the dimensionless group parameters. The Homotopy analysis method (HAM) contains the convergence control parameter h_1, so that it can be extend to solve the other MHD fluid flow problems in engineering and sciences.

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Appendix A

Basic concepts of the Homotopy analysis method [15-33]

Consider the following differential equation:

$$N[u(t)] = 0$$

(A.1)

Where N is a nonlinear operator, $t$denotes an independent variable, $u(t)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao (2012) constructed the so-called zero-order deformation equation as:

$$(1-p)\,L[\varphi(t;p)-u_0(t)] = ph\,H(t)\;N[\varphi(t;p)]$$

(A.2)

where $p\in[0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operator, $u_0\,(t)$ is an initial guess of $u(t)$, $\varphi(t:p)$ is an unknown function. It is important to note that one has great freedom to choose auxiliary unknowns in HAM. Obviously, when $p=0$ and $p=1$, it holds:

$$\varphi(t;0) = u_0(t) \text{ and } \varphi(t;1) = u(t)$$

(A.3)

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respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_0(t)$ to the solution u (t). Expanding $\varphi(t;p)$ in Taylor series with respect to p, we have:

$$\varphi(t;p) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t) p^m$$

(A.4)

Where

$$u_m(t) = \frac{1}{m!} \frac{\partial^m \varphi(t;p)}{\partial p^m}\Big|_{p=0}$$

(A.5)

If the auxiliary linear operator, the initial guess, the auxiliary parameter h, and the auxiliary function are so properly chosen, the series (A.4) converges at p =1 then we have:

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t).$$

(A.6)

Differentiating (A.2) for m times with respect to the embedding parameter p, and then setting p = 0 and finally dividing them by $m!$, we will have the so-called m th -order deformation equation as:

$$L[u_m - \chi_m u_{m-1}] = hH(t)\Re_m(\vec{u}_{m-1})$$

(A.7)

Where

$$\Re_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\varphi(t;p)]}{\partial p^{m-1}}$$

(A.8)

and

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases}$$

(A.9)

Applying L^{-1} on both side of equation (A.7), we get

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$$u_m(t) = \chi_m u_{m-1}(t) + hL^{-1}[H(t)\Re_m(\vec{u}_{m-1})]$$

(A.10)

In this way, it is easily to obtain u_m for $m \geq 1$, at M^{th} order, we have

$$u(t) = \sum_{m=0}^{M} u_m(t)$$

(A.11)

When $M \to +\infty$, we get an accurate approximation of the original equation (A.1). For the convergence of the above method we refer the reader to Liao [28]. If equation (A.1) admits unique solution, then this method will produce the unique solution.

Appendix B

Approximate analytical expressions of the non-linear differential eqns. (6) and (7) using Homotopy analysis method

In this Appendix, we indicate how the eqns. (19) - (20) are derived in this paper. To find the solution of the eqns. (6) and (7) when $\varepsilon\,\theta$ small, eqns. (6) and (7) reduces to

$$\frac{d^2w}{d\eta^2} - \varepsilon\frac{d\theta}{d\eta}\frac{dw}{d\eta} - (1+\varepsilon\theta)\left(Re\frac{dw}{d\eta} + Ha\,w - G\right) = 0 \quad \text{(B.1)}$$

$$\frac{d^2\theta}{d\eta^2} - Re\,Pr\frac{d\theta}{d\eta} + Ec\,Pr(1-\varepsilon\theta)\left(\frac{dw}{d\eta}\right)^2 + Ec\,Pr\,Ha\,w^2 = 0$$

(B.2)

We construct the Homotopy analysis method for the eqns (B.1) and (B.2) are as follows:

$$(1-p)\left(\frac{d^2w}{d\eta^2} + G\right) - h_1\,p\begin{pmatrix}\dfrac{d^2w}{d\eta^2} - \varepsilon\dfrac{d\theta}{d\eta}\dfrac{dw}{d\eta} - Re\dfrac{dw}{d\eta} - Ha\;w + G - \varepsilon\theta Re\dfrac{dw}{d\eta} \\ +\varepsilon\theta G - \varepsilon\theta Ha\;w\end{pmatrix} = 0$$

(B.3)

$$(1-p)\left(\frac{d^2\theta}{d\eta^2} - Re\,Pr\frac{d\theta}{d\eta}\right) - h_1\,p\begin{bmatrix}\dfrac{d^2\theta}{d\eta^2} - Re\,Pr\dfrac{d\theta}{d\eta} + Ec\,Pr\left(\dfrac{dw}{d\eta}\right)^2 \\ -Ec\,Pr\,\varepsilon\theta\left(\dfrac{dw}{d\eta}\right)^2 + Ec\,Pr\,Ha\,w^2\end{bmatrix} = 0$$

(B.4)

The approximate solution of the eqns (B.3) and (B.4) are as follows:

$$w = w_0 + p\,w_1 + p^2\,w_2 + p^3\,w_3 + p^4\,w_4 + \cdots\cdots$$

(B.5)

$$\theta = \theta_0 + p\,\theta_1 + p^2\,\theta_2 + p^3\,\theta_3 + p^4\,\theta_4 + \cdots\cdots$$

(B.6)

Substituting the eqns. (B.5) and (B.6) into the eqns. (B.3) and (B.4), we get the following results,

$$(1-p)\left[\frac{d^2(w_0 + pw_1 + \cdots)}{d\eta^2} + G\right] - h_1\,p\left[\begin{array}{l}\dfrac{d^2(w_0 + pw_1 + \cdots)}{d\eta^2} \\ -\varepsilon\dfrac{d(\theta_0 + p\theta_1 + \cdots)}{d\eta}\dfrac{d(w_0 + pw_1 + \cdots)}{d\eta} \\ -Re\dfrac{d(w_0 + pw_1 + \cdots)}{d\eta} - Ha(w_0 + pw_1 + \cdots) \\ +G + \varepsilon(\theta_0 + p\theta_1 + \cdots)G \\ -\varepsilon(\theta_0 + p\theta_1 + \cdots)Re\dfrac{d(w_0 + pw_1 + \cdots)}{d\eta} \\ -\varepsilon(\theta_0 + p\theta_1 + \cdots)Ha(w_0 + pw_1 + \cdots)\end{array}\right] = 0$$

(B.7)

$$(1-p)\left[\begin{array}{l}\dfrac{d^2(\theta_0 + p\theta_1 + \cdots)}{d\eta^2} \\ +Re\,Pr\dfrac{d(\theta_0 + p\theta_1 + \ldots)}{d\eta}\end{array}\right] - h_1\,p\left[\begin{array}{l}\dfrac{d^2(\theta_0 + p\theta_1 + \cdots)}{d\eta^2} - Re\,Pr\dfrac{d(\theta_0 + p\theta_1 + \cdots)}{d\eta} \\ +Ec\,Pr\left(\dfrac{d(w_0 + p\,w_1 + \cdots)}{d\eta}\right)^2 \\ -Ec\,Pr\,\varepsilon(\theta_0 + p\theta_1 + \cdots)\left(\dfrac{d(w_0 + p\,w_1 + \cdots)}{d\eta}\right)^2 \\ +Ec\,Pr\,Ha\,(w_0 + p\,w_1 + \cdots)^2\end{array}\right] = 0$$

(B.8)

Comparing the coefficients of p^0, p^1 in the eqns. (B.7) and (B.8) we get

$$p^0 : \frac{d^2 w_0}{d\eta} + G = 0$$

(B.9)

$$p^0 : \frac{d^2\theta_0}{d\eta} - Re\,Pr\frac{d\theta_0}{d\eta} = 0$$

(B.10)

$$p^1 : \frac{d^2 w_1}{d\eta^2} - \frac{d^2 w_0}{d\eta^2} - G - h_1 \begin{bmatrix} \frac{d^2 w_0}{d\eta^2} - \varepsilon \frac{d\theta_0}{d\eta}\frac{dw_0}{d\eta} - Re\frac{dw_0}{d\eta} - Ha\, w_0 - G \\ -\varepsilon\theta_0\, Re\frac{dw_0}{d\eta} - \varepsilon\theta_0\, Ha\, w_0 + \varepsilon\theta_0\, G \end{bmatrix} = 0$$

(B.11)

$$p^1 : \frac{d^2\theta_1}{d\eta^2} - Re\,Pr\frac{d\theta_1}{d\eta} - \frac{d^2\theta_0}{d\eta^2} + Re\,Pr\frac{d\theta_0}{d\eta} - h_1 \begin{bmatrix} \frac{d^2\theta_0}{d\eta^2} - Re\,Pr\frac{d\theta_0}{d\eta} \\ + Ec\,Pr\left(\frac{dw_0}{d\eta}\right)^2 - Ec\,Pr\,\varepsilon\theta_0\left(\frac{dw_0}{d\eta}\right)^2 \\ + Ec\,Pr\,Ha\,(w_0)^2 \end{bmatrix} = 0$$

(B.12)

The boundary conditions are as follows:

$$\left.\begin{aligned} w_0(0) = 0,\ \frac{d\theta_0}{d\eta}(0) = Bi_0(\theta_0(0) - 1), \\ w_0(1) = 0,\ \frac{d\theta_0}{d\eta}(1) = -Bi_1\,\theta_0(1). \end{aligned}\right\}$$

(B.13)

$$\left.\begin{aligned} w_j(0) = 0,\ \frac{d\theta_j}{d\eta}(0) = Bi_0\theta_j(0), \\ w_j(1) = 0,\ \frac{d\theta_j}{d\eta}(1) = -Bi_1\,\theta_j(1) \end{aligned}\right\}$$

(B.14)

Where $j = 1,2,3,\cdots$

By using the conditions (B.13) and (B.14) to Solving the eqns. (B.9)-(B.12)we can get the following results:

$$w_0 = \frac{-G\eta^2}{2} + \frac{G}{2}\eta \quad (B.15)$$

$$w_1 = -h_1 \left(\begin{array}{l} \dfrac{-c_2}{Re\,Pr}\left[\dfrac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr} - \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right] + \dfrac{c_2\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^3} + \dfrac{c_2\, G e^{Re\,Pr\,\eta}}{2(Re\,Pr)^2} - \dfrac{Re\,G\eta^3}{6} + \dfrac{Re\,G\eta^2}{4} \\ -\dfrac{Ha\,G\eta^4}{24} + \dfrac{Ha\,G\eta^3}{12} + \dfrac{c_3\eta^3}{6} - \dfrac{c_3\eta^2}{4} + \dfrac{c_4\eta^4}{12} - \dfrac{c_4\eta^3}{6} + \dfrac{c_5\,G\eta^2}{2} + \dfrac{c_6\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^3} \\ -\dfrac{c_6}{Re\,Pr}\left[\dfrac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr} - \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right] + \dfrac{c_6\, e^{Re\,Pr\,\eta}}{2(Re\,Pr)^2} + \dfrac{2c_7}{(Re\,Pr)^2}\left[\dfrac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr} - \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right] \\ -\dfrac{c_7}{Re\,Pr}\left[\dfrac{\eta^2 e^{Re\,Pr\,\eta}}{Re\,Pr} - \dfrac{2\eta\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^2} + \dfrac{2\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^3}\right] - \dfrac{2c_7\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^4} - \dfrac{c_7\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^3} \\ +\dfrac{c_7}{Re\,Pr}\left[\dfrac{\eta e^{Re\,Pr\,\eta}}{Re\,Pr} - \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right] - \dfrac{c_8\, e^{Re\,Pr\,\eta}}{(Re\,Pr)^2} - (c_9 + c_{10})\eta + c_9 \end{array} \right)$$

(B.16)

$$\theta_0 = -c_1\varepsilon\left(e^{Re\,Pr}(Re\,Pr + Bi_1) - Bi_1 e^{Re\,Pr\,\eta}\right)$$

(B.17)

$$\theta_1 = -h_1 \left(\begin{array}{l} c_{11} + c_{12}\, e^{Re\,Pr\,\eta} + \dfrac{Ec\,Pr}{Re\,Pr} \left[\begin{array}{l} \dfrac{G^2\eta}{4} + \dfrac{G^2\eta^3}{3} + \dfrac{G^2\eta^2}{Re\,Pr} + \dfrac{2G^2\eta}{(Re\,Pr)^2} - G^2\eta^2 - \dfrac{2G^2\eta}{Re\,Pr} \\ + \varepsilon c_1\, e^{Re\,Pr}(Re\,Pr + Bi_1)\left[\begin{array}{l} \dfrac{G^2\eta}{4} + \dfrac{G^2\eta^3}{3} + \dfrac{G^2\eta^2}{Re\,Pr} + \\ \dfrac{2G^2\eta}{(Re\,Pr)^2} - G^2\eta^2 - \dfrac{2G^2\eta}{Re\,Pr} \end{array} \right] \\ -3\,\varepsilon c_1 Bi_1 G^2 \dfrac{e^{Re\,Pr\,\eta}}{4\,Re\,Pr} - 2\,\varepsilon c_1 Bi_1 G^2 \dfrac{e^{Re\,Pr\,\eta}\eta^2}{Re\,Pr} \\ + 4\varepsilon c_1 Bi_1 G^2 \dfrac{e^{Re\,Pr\,\eta}\eta}{(Re\,Pr)^2} - 4\varepsilon c_1 Bi_1 G^2 \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^3} \\ + \left(4\varepsilon c_1 Bi_1 G^2 - 2\varepsilon c_1 Bi_1 \dfrac{G^2}{Re\,Pr}\right)\left[\dfrac{e^{Re\,Pr\,\eta}\eta}{Re\,Pr} - \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2}\right] \\ + 2\varepsilon c_1 Bi_1 G^2 \dfrac{e^{Re\,Pr\,\eta}}{(Re\,Pr)^2} \end{array} \right] \\ + \dfrac{Ec\,Pr\,Ha}{Re\,Pr} \left[\begin{array}{l} \dfrac{G^2\eta^3}{12} + \dfrac{G^2\eta^5}{20} - \dfrac{G^2\eta^4}{8} + \dfrac{G^2\eta^2}{4\,Re\,Pr} + \dfrac{G^2\eta^4}{4\,Re\,Pr} \\ - \dfrac{G^2\eta^3}{2\,Re\,Pr} + \dfrac{2G^2\eta}{4(Re\,Pr)^2} + \dfrac{G^2\eta^3}{(Re\,Pr)^2} - \dfrac{3G^2\eta^2}{2(Re\,Pr)^2} \end{array} \right] \end{array} \right)$$

(B.18)

Where $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}$ are defined in the eqns. (25)-(39) respectively.

According to the HAM, we can conclude that

$$w = \lim_{P \to 1} w(\eta) = w_0 + w_1$$
(B.19)
$$\theta = \lim_{P \to 1} \theta(\eta) = \theta_0 + \theta_1$$
(B.20)

After putting (B.15) and (B.16) into (B.19), (B.17) and (B.18) into (B.20) we obtain the solution in eqns. (19) and (20).

Appendix C

Nomenclature

Symbol	Meaning
U	Fluid velocity
C_P	Specific heat at a constant pressure
u	Fluid velocity
V	Uniform suction/injection velocity
T	Fluid temperature
T_f	Hot fluid temperature
h	Channel width
Ω	Temperature difference parameter
η	Dimensionless transverse coordinate
$\overline{\mu}(T)$	Temperature dependent viscosity
μ_0	Fluid viscosity at ambient temperature
μ	Fluid viscosity
Bi_1	Upper plate Biot number
Φ	Irreversibility ratio
ρ	Fluid density
σ	Electrical conductivity
γ_0	Lower plate heat transfer coefficient
γ_1	Upper plate heat transfer coefficient
T_∞	Ambient temperature
G	Pressure gradient
(x, y)	Cartesian coordinates
X	Dimensionless axial coordinate
m	Variable viscosity parameter
K	Thermal conductivity
P	Fluid pressure
Ha	Square of Hartmann number
$\Pr$	Prandtl number
Ec	Eckert number
W	Dimensionless velocity
α	Thermal diffusivity
θ	Dimensionless temperature