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PROPAGATION OF WEAK DISCONTINUITIES ALONG THE BICHARACTERISTIC CURVE IN A CHEMICALLY REACTING GAS

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Abstract

The transport equations representing the rate of change of discontinuities in the normal derivatives of the flow variables are obtained along bicharacteristic curves in the characteristic manifold of the differential equations governing the flow of a chemically reacting gas. The propagation of these weak discontinuities is discussed in particular cases of plane, cylindrical and spherical geometry. The effects of the chemical reactions, the magnitude of initial discontinuity, the initial curvature of the wave front and the upstream flow Mach number on the propagation of these weak discontinuities are investigated.

Key Words: Transport Equations, Bicharacterisitic Curves, Flow Variables.

Introduction

The behavior and propagation of weak discontinuities have been extensively studied in diverse branches of continuum mechanics (Thomas (1957), Truesdell and Toupin (1960), Thomas (1961), Varley (1965), Chen (1968), Sharma (1968). Nariboli and Secrest (1967), Ojha and Singh (1987, 1989), Ojha and Tiwari (1995, 1995) have discussed the growth and decay of these discontinuities in magnetogasdynamics and radiating gases. An account of real gas effects such as rotation, vibrational excitation. dissociation, ionization, chemical reaction etc. on the discontinuity surfaces of plane, cylindrical, and spherical geometries has been given by Becker(1968,1972)

By using the analytical method of characteristic, Clarke (1976, 1984) studied the progress of plane finite amplitude weak discontinuity through a reacting atmosphere. He has shown that this disturbance propagates through a uniform chemically reacting mixture of gas at the frozen speed of sound and if the degree of disequilibrium is sufficient it is amplified by a chemical reaction. The growth and decay of a weak discontinuity headed by a singular surface of arbitrary shape in three dimensions is investigated by Ojha, Tiwari and Takhar (1995) in a chemically reacting atmosphere in absence of dissipative mechanisms and the combined effects of the

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disequilibrium due to the chemical reaction and a wave front curvature on the propagation of discontinuities have been examined and discussed.

In the theory of hyperbolic systems of partial differential equations in four independent variables (x,y,z,t), the characteristic surfaces play an important role. One of the important properties of a characteristic surface is that discontinuities in the derivatives of the dependent variables can exist only along the characteristic surfaces. At any point of a characteristic surface there is a privileged direction given by the tangent of the bicharacteristic curve passing through that point. Making use of the results of differential geometry, Coburn (1957) derived an equation containing derivatives only along the bicharacteristic curve from the system of equations representing steady supersonic flow of an inviscid and non-conducting gas. Gubkin (1958) making assumption of short `waves, derived an approximate quasilinear equation giving the rate of change of the amplitude of a weak disturbances in the bicharacteristic direction. This work was further generalized by Prasad (1975) for a general system of partial differential equations.

It has been shown by Elcrat (1977) that if discontinuity surface is adjacent to a region of uniform flow it is necessary to consider the transport of discontinuities along the characteristic curves in the characteristic manifold of the differential equations. Sharma and Shyam (1982) have obtained the transport equation representing the rate of change of discontinuities in the normal derivatives of flow parameters along the bicharacteristic curve in the characteristic manifold of the differential equations governing the flow of a vibrationally relaxing gas.

Our aim here is to derive transport equations for the discontinuities in the normal derivatives of flow parameters along the bicharacteristic curves in the characteristic manifold of differential equations governing an unsteady flow of a chemically reacting gas. It is observed that the transport equations along bicharacteristics reduce to the corresponding equations along orthogonal trajectories when the singularity surface is adjacent to a region of rest. In particular, the growth and decay of weak discontinuities along bicharacteristics are discussed in cases of plane, cylindrical and spherical geometries.

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2. Basic Equations

The differential equations governing an unsteady flow of a chemically reacting gas are (Ojha et.al.(1995)),

(2.1) $$\frac{\partial\rho}{\partial t} + u_i \rho_{,i} + \rho u_{i,i} = 0$$

(2.2) $$\rho \frac{\partial u_i}{\partial t} + \rho u_j u_{i,j} + p_{,i} = 0$$

(2.3) $$\rho\left(\frac{\partial h}{\partial t} + u_i h_{,i}\right) = \frac{\partial p}{\partial t} + u_i p_{,i}$$

(2.4) $$\frac{\partial q}{\partial t} + u_{i,} q_{,i} = W$$

The summation convention on repeated indices is employed and a comma followed by an index denotes a partial derivative with respect to a space variable. The range of Latin indices is taken to be 1, 2, 3; other symbols appearing in these equations are as follows: ρ is density; p is pressure; u_i are the components of the reacting gas velocity; q is the chemical progress variable representing the mass fraction of the reactant species F per unit mass which takes part in a simple irreversible reaction of n^{th} order.

(2.5) nF = P

i.e. n molecules of F combine to create a product species P; h is the enthalpy per unit mass of the gas mixture defined by

(2.6) $$h = e + \frac{p}{\rho}$$

where $e = e(p, \rho, q)$ is the internal energy per unit mass of the gas mixture; W=W (p,ρ,q) is the reaction rate or rate of progress of reaction, defined by Clarke (1984),

where $$W = -\frac{nw}{Z_Z}\exp\left(-\frac{E_A}{RT}\right)q^n = -\frac{nw}{Z}q^n$$

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$$Z = Z_Z \exp\frac{E_A}{RT} = Z_z \exp\left(\frac{\rho\bar{\theta}}{p}\right)$$

here , w is the molecular weight of the species F; E_A is activation energy; R is the gas constant; T is the absolute temperature; Z_Z has the dimension of time which is a weak function of T and p and is often treated roughly as constant; $\bar{\theta}$ defined here is usually compared with p/ρ and is a constant parameter. Denoting the ratio of specific heats of the gas mixture under chemically frozen condition by γ_f, which is not necessarily constant, the frozen speed of sound a_f is defined by

(2.8) $$a_f^2 = \frac{\gamma_f p}{\rho}$$

Equation (2.3) with the use of (2.1), (2.2), (2.3), (2.4) and (2.6) - (2.8) is conveniently transformed to (Clarke and Mcchesney (1976),Clarke (1977)),

(2.9) $$\frac{\partial p}{\partial t} + u_i p_{,i} + \rho a_f^2 u_{i,i} + \rho\left(\gamma_f - 1\right)QW = 0$$

Q is the energy of formation per unit mass of reactant species.

3. Velocity of Propagation

Equations (2.1), (2.2), (2.4) and (2.9) can be written as in form

(3.1) $$U_i + A^i U_{,i} + B = 0$$

where $$U_{,t} = \frac{\partial U}{\partial t}$$

The set U is a column matrix with six componentsρ, u_i, q and p. A^i are the square matrix of order six and B is a column matrix, which can be read off by inspection of (2.1), (2.2), (2.4) and (2.9).

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Consider a characteristic wave surface Σ, $\phi(x_1, x_2, x_3, t) = 0$ across which the set of dependent variables U and the interior derivatives of U along Σ are continuous but the exterior derivatives of U may have a jump across it.

The jump in a quantity f is denoted by [f]; the quantity f may represent any of the parameters ρ, u_i, q and p. The square bracket stands for the value of the quantity enclosed immediately behind the wave surface minus its value just ahead of the wave surface. Since the parametersρ, u_i, q and p are essentially continuous across Σ, we infer that the quantities a_f and W are also continuous across Σ and that they will have their sub-scripts-o values at the wave front where the subscript-o indicates a value in the region just ahead of the wave surface Σ. The medium ahead of the wave is assumed to be uniform and in equilibrium, we have

(3.2) $$W_o = 0$$

Introduce new co-ordinates $\xi^o, \xi^1, \xi^2, \xi^3$ to be chosen such that ξ^1, ξ^2, ξ^3 determine a point on Σ which is itself defined by $\phi = 0$ and set $\xi^o = \phi$. In terms of these new coordinates, the system (3.1) can be written as

(3.3) $$\left(I\phi_{,t} + A^i\phi_{,i}\right)U_{,\phi} + A^i\xi^f_{,i}U_{,\xi j} = 0$$

where I is the unit matrix of order six. Forming the jumps in the usual way we obtain from (3.3),

(3.4) $$\left(I\phi_{,t} + A^i\phi_{,i}\right)\left[U_{,\phi}\right] = 0$$

This follows because the interior derivatives $U_{,\xi}j$ are continuous across Σ; the discontinuities existing only in the exterior derivatives $U_{,\phi}$. Dividing equation (3.4) by $\left(\phi_{,i}\phi_{,i}\right)^{1/2}$ and setting $\frac{\phi_{,i}}{\left(\phi_{,j}\phi_{,j}\right)^{1/2}} = n_i$ (the unit normal to the wave surface Σ) and $\frac{\phi_{,t}}{\left(\phi_{,j}\phi_{,j}\right)^{1/2}} = -G$ (the speed of propagation of Σ along n_i,)the system (3.4) will have a non-trivial solution if

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(3.5) $\left| A^i n_i - GI \right| = 0$

which yields

(3.6) $G - u_{n_o} = \pm a_{f_o}$

the speed of wave front Σ relative to the fluid

where

(3.7) $u_{n_o} = u_{i_o} n_i$

Considering the case of advancing disturbance only, we take

(3.8) $G = u_{n_0} + a_{f_0}$

The order of the characteristic matrix of the system (3.4) is equal to six, its rank is equal tom five, when (3.5) holds. Thus for $G = u_{n_0} + a_{f_0}$ the characteristic matrix has an eigen vector, the components of which are(1, a_{f_0} , n_{i/ρ_o} , 0, $a^2_{f_0}$) . This means that the jumps in the normal derivatives of the ρ, $u_{i,}$ q and p denoted by $\Im$,λ_i ,ν and p respectively ,are given by

(3.9) $\Im = \frac{\rho_0}{a_{f_0}} = \frac{\xi}{a^2_{f_0}} = \sigma$ and $\nu = 0\, \Im = \frac{\rho_0}{a_{f_0}} = \frac{\xi}{a^2_{f_0}} = \sigma$

where $\lambda = \lambda_i n_i$ and σ is a non- zero scalar.

4. Wave Geometry and Bicharacteristic

Let us recall that a bicharacteristic curve $x_i = x_i(t)$ for the system (2.1), (2.2), (2.4) and (2.9) satisfies the equation (Courant and Hilbert (1962))

(4.1) $$\frac{\hat{d}x_i}{dt} = \frac{\partial F}{\partial \phi_{,i}} \Big/ \frac{\partial F}{\partial \phi_{,t}}$$

where $\frac{\hat{d}}{dt}$ is the time derivative moving with the wave front along the bicharacteristic curve defined as

(4.2) $$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_{,i}}$$

(v_i are the bicharacteristic velocity components) and $F = \left| I\phi_{,t} + A^i \phi_{,i} \right| = 0$ is the characteristic equation for the system which for the present case takes the form

(4.3) $$F = \phi_{,t} + u_{,i}\phi_{,i} + a_{f_0}\left(\phi_{,i}\phi_{,i}\right)^{1/2}$$

Equation (4.1) together with (4.3) yields

(4.4) $$v_i = \frac{\hat{d}x_i}{dt} = u_i + a_{f_0} n_i$$

we define $\frac{\delta}{\delta t}$ to be the time rate of change moving with the wave front along the normal n_i as follows

(4.5) $$\frac{\delta}{\delta t} = \frac{\partial}{\partial t} + G n_i \frac{\partial}{\partial x_i}$$

which is δ-derivative of Thomas (1961).Equations (4.2), (4.4) and (4.5) can be combined to yield

(4.6) $$\frac{\hat{d}}{dt} = \frac{\delta}{\delta t} + u^{\alpha} x_{i,\alpha} \frac{\partial}{\partial x_i}$$

where $x_{,\alpha} = \frac{\partial x_{,i}}{\partial y_{\alpha}}$ are the components of the projective tensor of the discontinuity surface Σ, $x_i = x_i(y^1, y^2, t)$; y^{α} (α=1,2) being the Gaussian co-ordinates on the surface Σ and $u^{\alpha} = u_i x_{i,\alpha}$ represent the velocity components along the discontinuity surface Σ.Thus, for any flow variable f defined on Σ, we have the relation

(4.7) $$\frac{\hat{d}f}{dt} = \frac{\delta f}{\delta t} + u^{\alpha} f_{,\alpha}$$

since $$x_{i,\alpha} f_{,i} = f_{,\alpha}$$

5. Derivation of the Growth Equation

Differentiating (2.2) and (2.9) with respect to x_i and applying the jump conditions across Σ(t), we obtain (Ojha et.al.(1995)).

(5.1) $$\rho\left[\frac{\partial^2 u_i}{\partial x_j \partial t}\right] + \left[\rho_{,j}\frac{\partial u_i}{\partial t}\right] + \left[u_{k,j} u_{i,k}\right] + \left[p_{,ij}\right] = 0$$

(5.2) $$\left[\frac{\partial^2 p}{\partial x_j \partial t}\right] + \left[u_{i,j} p_{,i}\right] + \left[\rho u_{i,i} a^2_{f,j}\right] + \rho a_f^2\left[u_{i,ij}\right] + \left[\rho_{,j}\left(\gamma_f - 1\right) QW\right]$$

$$+\rho\left(\gamma_f - 1\right)\left[\left(QW\right)_{,j}\right] = 0$$

Using the compatibility conditions of first and second order due to Thomas (1957) in equations (5.1) and (5.2), we have

(5.3) $$\rho_o\left(\frac{\delta\lambda}{\delta t} + u^{\alpha}\lambda_{,\alpha}\right) - \left(\bar{\xi} - \rho_o a_{fo}\bar{\lambda}\right) = 0$$

(5.4) $$\frac{\delta\xi}{\delta t} + u^{\alpha}\xi_{,\alpha} - \left(a_{fo}\,\bar{\xi} - \rho_o a_f^2\bar{\lambda}\right) + 2\left(\lambda_o - a_f^2\Omega\right)\lambda + \Gamma_{fo}\xi\lambda = 0$$

where $$\Gamma_f = \frac{1}{a_f}\frac{\partial}{\partial\rho}\left(\rho a_f\right)$$

(5.5) $$a_f\left(\Gamma_f - 1\right) = \rho\left[a_f^2\left(\frac{\partial a_f}{\partial} p\right)_{\rho,q} + \left(\frac{\partial a_f}{\partial \rho}\right)_{p,q}\right]$$

(5.6) $$\wedge = \frac{(\gamma f - 1)}{2}\left[\frac{QW}{a_f^2} + \rho\left\{\left(\frac{\partial(QW)}{\partial p}\right)_{p,q} + \frac{1}{a_f^2}\left(\frac{\partial(QW)}{\partial \rho}\right)_{p,q}\right\}\right]$$

(5.7) $$\bar{\lambda} = \left[u_{i,jk}\right] n_i n_j n_k$$

(5.8) $$\bar{\xi} = \left[p_{,ijk}\right] n_i n_j$$

and

(5.9) $$\Omega = \frac{1}{2} g^{\alpha\beta} b_{\alpha\beta}$$

$\wedge_o$ and Γ_{f_o} are the values of $\wedge$ and Γ_f ahead to the wave and Ω is the mean curvature of Σ with $g^{\alpha\beta}$ and $b_{\alpha\beta}$ being the first and second fundamental form of Σ respectively. Eliminating $\left(\bar{\xi} = \rho_0 a_{f_0} \bar{\lambda}\right)$ between (5.3) and (5.4), we get

(5.10) $$\rho_0 a_{f_0} \alpha\left(\frac{\delta\lambda}{\delta t} + u^\alpha \lambda_{,\alpha}\right) + \left(\frac{\delta\xi}{\delta t} + u^\alpha \xi_{,\alpha}\right) + 2\left(\wedge_0 - a_{f_0}\Omega\right)\lambda + \Gamma_{f_0}\xi\lambda = o$$

In equation (5.10), the term envolving surface derivatives cause and some difficulty in its interpretation but if we transform (5.10) to a differential equation along bicharacteristics, this difficulty disappears. This can be done easily if we make use of the relation (4.6) in (5.10). Thus eliminating the term ξ from (5.10) by using relation (3.9), the transport equation along bicharactristics for the quantity λ can be written as

(5.11) $$\frac{d\hat{\lambda}}{dt} + \left(\wedge_0 - a_{f_0}\Omega\right)\lambda + \Gamma_{f_0}\lambda^2 = 0$$

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Equation (5.11) is the basic differential equation for the growth and decay of weak discontinuities associated with surface $\Sigma(t)$ along the bicharacteristic curves that are propagating into the uniform region at rest with constant density, pressure and chemical progress variable. It is clear that the behavior of the solution of equation (5.11) will depend entirely on the sign of $\wedge_o$ and Ω. In absence of the chemical reaction, the growth equation is reduced to the growth equation obtained by Thomas (1958).

Integrating (5.11) we get

(5.12) $$\lambda = \frac{\lambda_0 A e^{-\wedge_0 t}}{1+\lambda_0 \Gamma_{f_0} B}$$

where

(5.13) $$A = \exp \int_o^t a_{f_0} \Omega dt$$

(5.14) $$B = \int_o^t A e^{-\wedge_0 t} dt$$

and λ_o is the value of λ at the wave front at time t = 0. In order to predict the physical aspect more clearly, we discuss the particular cases of plane, cylindrical and spherical waves. In these cases the bicharacteristic curves are, of course, straight lines emanating from the center of the wave propagation.

6. Discussions

(i) Plane Waves

For plane wave front Ω =0 and therefore solution of (5.11) reduces to

(6.1) $$\lambda = \frac{\lambda_o e^{-\wedge_o t}}{1+\frac{\lambda_o}{\lambda_c}\left(1-e^{-\wedge_o t}\right)}$$

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where

(6.2) $$\lambda_c = \frac{\wedge_0}{\Gamma_{f_0}}$$

Equation (6.1) shows that if $\lambda_0 > 0$ (i.e. an expansive wave front) then $\lambda \to 0$ as $t \to \infty$ and the wave decays. Also if $\lambda_0 < 0$ (i,e. a compressive wave front) and it has the magnitude less than λ_c, then $\lambda \to 0$ as $t \to \infty$ i.e. a compressive wave decays and ultimately disappears. Further if $\lambda_0 = -\lambda_c$ then $\lambda = \lambda_0$ and the wave attains a stable wave form. But if λ_0 is negative and has a magnitude greater than λ_c, then λ increases beyond all bounds for a finite time t approaching the value

(6.3) $$t_c = \frac{1}{\wedge_0} \log\left(1 + \frac{\wedge_0}{\lambda_0 \Gamma_{f_0}}\right)$$

Thus at the instant t_c , the velocity gradient at the wave front becomes infinite and this signifies the appearance of shock wave. The initial value of the discontinuity $\lambda_0 = -\lambda_c$ plays an important role in determining whether the wave shall grow or decay. We say this value of λ_0 is the critical value of λ_0 for a plane wave.

From (6.3) it follows that $\frac{\partial t_c}{\partial\left(\wedge_0 / \Gamma_{f_0}\right)} \succ 0$ i.e. an increase in $\frac{\wedge_0}{\Gamma_{f_0}}$ causes an increase in t_c. Thus, the chemical reaction has a delaying effect on the formation of shock wave. The effect of chemical reaction causes an expansive wave of given initial discontinuity to decay at a faster rate relative to what it would be in the absence of chemical reaction.

(ii) Cylindrical waves

Consider an outward travelling wave front at time t = 0 is a cylinder of radius R_o. Then at any later time the wave front is a cylinder of radius $R = R_o + Gt$. For such a wave

$\Omega = -Q'/2R$, $u_{n_0} = u_0$, the radial velocity ahead of the wave and thus $G = a_{f_o}(M_0 + 1)$ where $M_0 = u_o/a_{f_o}$ the upstream flow Mach number. Then (5.11) in the two case reduces to

(6.4) $$\frac{d\hat{\lambda}}{dt} + \left(\lambda_0 + \frac{a_{f_0}}{2R}\right)\lambda + \Gamma_{f_0}\lambda^2 = 0$$

Integrating (6.4), we have

(6.5) $$\lambda = \frac{\lambda_o e^{(\mu_o - \mu)}\left(R/R_o\right)^{-n}}{1 - \frac{\lambda_o \Gamma_{f_o}}{\wedge_o}\mu_o^n e^{\mu_o} E(\mu_o)\left(1 - \frac{E(\mu)}{E(\mu_o)}\right)}$$

where

(6.6) $$\frac{1}{M_o + 1} = 2n > 0 \qquad \mu = \frac{2n \wedge_o R}{a_{fo}} \qquad \mu_o = \frac{2n \wedge_o R_o}{a_{fo}}$$

and

(6.7) $$E(\mu) = \int_x^\infty \mu^{-n} \exp(-\mu) du$$

Is an improper integral convergent for positive values of n and x considered here,

Writing $\bar{\lambda}_c = \dfrac{1}{\frac{F_{fo}}{\wedge_o}\mu_o^n e^{\mu_o} E(\mu_o)}$ we have

(6.8) $$\lambda = \frac{\lambda_o e^{(\mu_o - \mu)}\left(\frac{R}{R_o}\right)^{-n}}{1 - \frac{\lambda_o}{\lambda_c}\left(1 - \frac{E(\mu)}{E(\mu_o)}\right)}$$

The term inside the bracket in the denominator of (6.8) increases monotonically from 0 to 1 as R increases from $R_0 \to \infty$. Hence if $\lambda_o > 0$ or if $\lambda_o < 0$ and $|\lambda_o| < \bar{\lambda}_c$ then it follows from

(6.8) that $\lambda \rightarrow 0$ as $R \rightarrow \infty$, the wave decays. However, if $\lambda_o < 0$ and $|\lambda_o| = \bar{\lambda}_c$ then at any time t, the discontinuity I λ I is given by

(6.7) $$|\lambda| = \frac{\wedge_o}{\Gamma_{fo}} \frac{e^{-\mu}\left({R}/{R_o}\right)^{-n}}{\mu_o^n E(\mu)}$$

which approaches the critical values $\frac{\wedge_o}{\Gamma_{f_o}}$ (critical λ_o for plane wave) as $R \rightarrow \infty$, i.e. the wave takes the stable wave form. But if $\lambda_o < 0$ and $|\lambda_o| = \bar{\lambda}_c$ then there exists a finite time t_c given by

(6.8) $$E(\mu) = \left(1 - \frac{\bar{\lambda}_c}{|\lambda_o|}\right) E(\mu_o)$$

such as $|\lambda| \rightarrow \infty$ as $t \rightarrow \bar{t}_c$ .This signifies the appearance of the shock wave at a finite time $\bar{t}_c$. Equation (6.8) shows that the shock formation time t_c depends on λ_o, $\frac{\wedge_o}{\Gamma_{fo}}$, M_o and R_o. It follows from (6.8) that $\frac{\partial \bar{t}_c}{\partial R_o} < 0$ which mean that an increase in the initial curvature will cause the shock formation time $\bar{t}_c$ to increase. Also $\frac{\partial \bar{t}_c}{\partial |\lambda_o|} < 0$ means that an increase in the initial value of the discontinuity leads to a rapid shock formation.

(iii) Spherical waves

Consider an expanding wave front which at t = 0 is a sphere of radius R_0. At any time t > 0,the wave front is a sphere of radius $R = R_0 + Gt$. For such a wave. For such a wave $\Omega = -\frac{1}{R}$. In this case, the growth and decay equation (5.11) reduces to

(6.9) $$\frac{d\hat{\lambda}}{dt} + \left(\wedge_0 + \frac{a_{f_0}}{R}\right)\lambda + \Gamma_{f_0}\lambda^2 = 0$$

Integrating (6.9), we have

(6.10) $$\lambda = \frac{\lambda_o \left(\frac{R}{R_o}\right)^{-2n} e^{(\mu_o - \mu)}}{1 - \frac{\lambda_o}{\lambda_c}\left(1 - \frac{E_1(\mu)}{E_1(\mu_o)}\right)}$$

where

(6.11) $$E_1(\mu) = \int_x^\infty \exp(-\mu).\mu^{-2n} du$$

and

(6.12) $$\bar{\lambda}_c = \frac{\wedge_o}{\Gamma_{f_o} e^{\mu_o} E_1(\mu_o)\mu_o^{2n}}$$

It follows from (6.10) that for $\lambda_0 < 0$ and $\lambda_o < \lambda_c$ waves damp to zero and waves whose initial discontinuity is greater than λ_c grow without bound in a finite time $\bar{t}_c$ given by

(6.13) $$E_1(\mu) = \left(1 - \frac{\bar{\lambda}_c}{\lambda}\right) E(\mu_o)$$

For $\lambda_0 \prec 0$ and I $|\lambda_o| = \bar{\lambda}_c$ it follows from (6.10) that the wave can neither terminate into a shock nor ever completely damp out. In addition, it follows from (6.10) that $|\lambda| \to \frac{\wedge_o}{\Gamma_{f_o}}$ (critical λ_0 for a plane wave) as $R \to \infty$. Thus a compressive wave with I λ_0I = λ_c takes ultimately a stable wave form.

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